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Creation of entanglement between two electron spins induced by many spin ensemble excitations

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Abstract. We theoretically explore the possibility of creating spin entanglement by simultaneously coupling two electronic spins to a nuclear ensemble. By microscopically modeling the spin ensemble as a single mode boson field, we use the time-dependent Fröhlich transformation (TDFT) method developed recently [Y. Li, C. Bruder, C.P. Sun, Phys. Rev. A **75**, 032302 (2007)] to calculate the effective coupling between the two spins. Our investigation shows that the total system realizes a solid state based architecture for cavity QED. Exchanging such kind of effective boson in a virtual process can result in an effective interaction between two spins. It is discovered that a maximum entangled state can be obtained when the velocity of the electrons matches the initial distance between them in a suitable way. Moreover, we also study how the number of collective excitations influences the entanglement. It is shown that the larger the number of excitation is, the less the two spins entangle each other.

PACS. 68.65.Hb Quantum dots – 03.67.Mn Entanglement production, characterization, and manipulation – 73.21.-b Electron states and collective excitations in multilayers, quantum wells, mesoscopic, and nanoscale systems

1 Introduction

Since Shor and Grover algorithms [1,2] were proposed with various following significant developments, e.g., [3], quantum computing has been displaying its more and more amazing charm against classical computing. As more progress has been made in this area, it is urgent to discover robust, controllable and scalable two-level systems — qubits as the basic elements for the future architecture of quantum computers. Generally speaking, electron spins are a natural qubit, especially the single electron spin confined in a quantum dot for its well separation and easy addressability. In reference [4], electron spins in quantum dots were employed as qubits and two-qubit operations were performed by pulsing the electrostatic barrier between neighboring spins. Thereafter, Kane's model made use of the nuclear spins of ^{31}P donor impurities in silicon as qubits [5]. It combined the long decoherence time of nuclear spins and the advantage of the well developed modern semiconductor industry.

In practice, it seems difficult to control the coupling between qubits because the coupling is based on the overlap of two adjacent spin wave functions [4,5]; the coupling is given to be fixed once photolithography of the chip has been finished. Another feasible way to induce the controllable inter-spin interaction is to couple two spins by spin-orbit interaction [6]. However, because of the weakness of spin-orbit coupling, it is very crucial to find a scheme to manipulate two spin coupling in the strong interaction regime. In the present paper, we consider the possibility of creating quantum entanglement of two electron spins by making them pass through a 2D quantum well containing many "cooled" nuclear spins (see Fig. 1). It was discovered that the effective coupling intensity was increased by a factor of $\sqrt{N}$ when an electron spin was coupled with an ensemble of N nuclear spins [7]. Such an electron spin coupled to the nuclei has been considered for cooling the nuclear ensemble [8]. Moreover, people have proposed a quantum computing scheme using a scanning tunneling microscopy with a moving tip as a commuter to perform the control-not gate between two qubits on the silicon surface [9,10]. Here, the tip played the role of the quantum data bus to coherently link the qubits.

Enlightened by these works, we suggested a new scheme to entangle two electron spins by a tip since it could be modeled as an ensemble of many spins [11]. Indeed, when the couplings of the spin to the nuclear ensemble are quasi-homogeneous, the interaction between

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the electron spin and the collective excitation of nuclear spins can be well described in terms of artificial cavity QED [12]. Here, the collective excitation can behave as a single mode boson to realize a quantum data bus, while the electron spin acts as a two-level artificial atom. With the frequency selection due to the resonance effect, there is only one mode of collective excitations interacting with the two spins. Especially, when the Zeeman splits of all nuclear spins are the same, the single mode excitation can decouple with other modes [12]. Then, the coupling system including two qubit spins and nuclear ensemble just acts as a typical cavity QED system or spin-boson system. To coherently manipulate the indirect interaction between the two spins, which is induced by the above mentioned collective excitation, we need to let two electrons go through the quantum well to realize a two qubit logical gate operation. Since the moving of electrons leads to a time-dependent coupling, we need to use some new method to derive the effective Hamiltonian for the inter-spin coupling. Fortunately, a recent paper suggested such a time-dependent approach [13].

The rest of our paper is organized as follows. In Section 2, in the low excitation limit, we simplify the total system we considered above as two spins interacting with a single mode of the collective excitation of the nuclei, which forms a cavity-QED under the quasihomogeneous condition. In Section 3, we derive the effective Hamiltonian between the two electron spins by the time-dependent Fröhlich transformation (TDFT) method developed recently in reference [13]. We remark that the TDFT method can be used to derive an effective Hamiltonian for a class of cavity QED systems with time-dependent perturbations. Here, we use this transformation for the case of time-dependent couplings of two spins to a many spin ensemble. Section 4 contains the discussion of entanglement induced by the effective Hamiltonian and collective excitation's effect on the entanglement. In Section 5 we review most of the significant results. Finally, technical details are given in Appendices A and B.

2 Model description

We consider a system illustrated in Figure 1. Two electrons go through a quantum well one after the other. The electron spins described by Gaussian packets with a width $a/2$ are initially located at $z_0^{(i)}$ $(i = 1, 2)$ and move along the z-direction with a uniform speed v. The 2D quantum well consists of many polarized nuclear spins located in position (x_j, y_j, z_j) with $|x_i| \leq a$, $|y_i| \leq a$, $|z_i| \leq a/5$. When a static magnetic field is applied to the total system, the Hamiltonian reads

$$\begin{aligned} H^S = {} & \Omega_z(S_z^{(1)} + S_z^{(2)}) + \omega_z \sum_i I_z^{(i)} \\ & + S_z^{(1)} \sum_i g_1^{(i)} I_z^{(i)} + S_z^{(2)} \sum_i g_2^{(i)} I_z^{(i)} \\ & + S_+^{(1)} \sum_i \frac{g_1^{(i)}}{2} I_-^{(i)} + S_+^{(2)} \sum_i \frac{g_2^{(i)}}{2} I_-^{(i)} + h.c., \quad (1) \end{aligned}$$

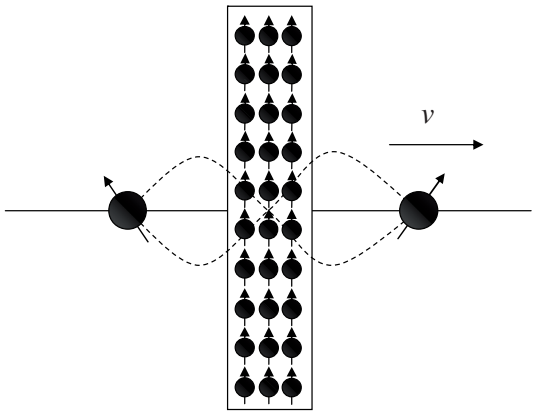

Fig. 1. Schematic diagram of two electron spins going through a quantum well consisting of nuclear spins. Two electrons initially located at $z_0^{(1,2)}$ move with a uniform speed v.

where $S_z^{(l)}$ and $S_\pm^{(l)}$ $(= S_x^{(l)} \pm iS_y^{(l)})$ $(l = 1, 2)$ are the spin operators for the lth electron spin, $I_z^{(j)}$ and $I_\pm^{(j)}$ $(= I_x^{(j)} \pm iI_y^{(j)})$ $(j = 1, 2, \cdots, N)$ the spin operators for the jth nuclear spin, $g_l^{(j)}$ $(l = 1, 2, j = 1, 2, \cdots, N)$ the hyperfine coupling constants between lth electron and jth nuclear spin. The first and second terms of Hamiltonian (1) are the Zeeman energies for the electron spins and the nuclear spins respectively, and the terms besides them are the hyperfine interaction between the electrons and nuclear spins.

In our setup, the nuclear spins are restricted in a flat square box with $|x_i| \leq a$, $|y_i| \leq a$, $|z_i| \leq a/5$. We have (for the necessary details please refer to Appendix A)

$$g_1^{(i)} \simeq g_0^{(i)} f_1(t), \;\; g_2^{(i)} \simeq g_0^{(i)} f_2(t). \quad (2)$$

In reference [12], the collective excitation of an ensemble of polarized nuclei fixed in a quantum dot was studied. Under the approximately homogeneous condition the many-particle system behaves as a single-mode boson interacting with the spin of a single conduction-band electron confined in this quantum dot. Likewise, we introduce a collective operator

$$B = \frac{\sum_{i=1}^N g_0^{(i)} I_-^{(i)}}{\sqrt{2I_0 \sum_j [g_0^{(i)}]^2}}$$

and its conjugate B^+ to depict the collective excitations in the ensemble of nuclei with spin I_0 from its polarized initial state

$$|G\rangle = \prod_{i=1}^N |-I_0\rangle_i$$

which is the saturated ferromagnetic state of nuclear ensemble. In our model, the nuclear spins are fixed in GaAs crystal lattice with $I_0 = 3/2$, $a = 4$ nm and the density of nuclei $n_0 = 45.6$ nm^{-3}. For simplicity, we assume the nuclei are located in a simple cubic lattice. Thus, we have the average distance between two neighboring nuclear spins $d = 0.28$ nm, $g_{max}^2/\overline{g^2} \simeq 11.7$, $N = 5046$, where

g_{max} is the maximum of $g_0^{(i)}$ and $\overline{g^2} = \sum_i (g_0^{(i)})^2/N$. On condition that the number of excitations in the system $n \ll NI_0\overline{g^2}/g_{max}^2$, we have $[B, B^+] \to 1$. In other words, the collective excitation described by B can behave as a boson mode in the large N limit with an initial polarization of all spins in ground (spin down) state.

In addition to the basic mode denoted by B and B^+, there exist auxiliary modes

$$C_k = \frac{\sum_{i=1}^N h_i^{[k]} I_-^{(i)}}{\sqrt{2I_0(\boldsymbol{h}^{[k]})^2}}$$

for $k = 1, 2, \ldots, N$. Here,

$$\boldsymbol{h}^{[k]} = (h_1^{[k]}, h_2^{[k]}, \ldots, h_N^{[k]})$$

are N orthogonal vectors in N-dimensional space $\mathbf{R}^N$, which can be systematically constructed by making use of the Gram-Schmidt orthogonalization method [14] starting from

$$\boldsymbol{h}^{[1]} = (g_0^{(1)}, g_0^{(2)}, \ldots, g_0^{(N)}) \in \mathbf{R}^N.$$

Therefore, the Hamiltonian (1) is rewritten as

$$\begin{aligned} H^S \simeq & (\Omega_z - f_1 I_0 \sum_i g_0^{(i)}) S_z^{(1)} + (\Omega_z - f_2 I_0 \sum_i g_0^{(i)}) S_z^{(2)} \\ & + f_1 \Omega (S_+^{(1)} B + S_-^{(1)} B^+) + f_2 \Omega (S_+^{(2)} B + S_-^{(2)} B^+) \\ & + \omega_z \sum_k C_k^+ C_k + \omega_z B^+ B + H_p^S. \end{aligned} \tag{3}$$

Here, the effective Rabi frequency

$$\Omega = \sqrt{\frac{1}{2} I_0 \sum\nolimits_{i=1}^N \left[g_0^{(i)}\right]^2}.$$

describes the enhanced coupling of the electron spin to the collective excitation. And the single particle excitation term

$$H_p^S = (S_z^{(1)} f_1 + S_z^{(2)} f_2) \sum_i g_0^{(i)} (I_z^{(i)} + I_0) \tag{4}$$

can be treated as a perturbation term in the low-excitation limit, which originates from the inhomogeneity of the couplings.

3 Effective inter-spin coupling description

As shown in Hamiltonian (3), there are only couplings of two electron spins with the single mode boson respectively. By making use of the canonical transformation [13], we can eliminate the boson operator and obtain the effective interaction between the two electron spins. Former research mainly focused on the case where $f_1(t)\Omega$ and $f_2(t)\Omega$ were time independent [15]. However,the time-independent approach may not work well in practice. Now, because of the motion of the electrons, we take the time-dependence of interaction into consideration, namely, $f_1(t)\Omega$ and $f_2(t)\Omega$ depend on time.

Let us first summarize the main idea of the time-dependent Fröhlich transformation [13] so that our paper is self consistent for reading. Generally speaking, Fröhlich transformation [16,17] is frequently used in condensed matter physics to obtain effective interaction between two electrons by exchanging virtual phonons. For a quantum system described by Hamiltonian $H(t) = H_0 + H_1(t)$, where H_0 is time independent and $|H_0| \gg |H_1(t)|$, we can make a canonical transformation

$$|\psi(t)\rangle \to e^{-F(t)}|\psi(t)\rangle, \quad H(t) \to e^{-F(t)} H(t) e^{F(t)}, \tag{5}$$

where $F(t)$ is an anti-Hermitian operator and $\psi(t)$ the state of the system. When $F(t)$ is appropriately chosen to make the first-order term of the effective Hamiltonian vanishing, i.e., $H_1 + [H_0, F] - i\partial_t F = 0$, we obtain an effective Hamiltonian to the second order $H_{\mathit{eff}} = H_0 + [H_1, F]/2$ in principle, and the above equation explicitly determines $F = F(t)$.

In this section, the canonical transformations are made to obtain the effective Hamiltonian. In the interaction picture with respect to

$$H_0^S = \omega_z (S_z^{(1)} + S_z^{(2)}) + \omega_z (B^+ B + \sum_k C_k^+ C_k),$$

the Hamiltonian $H^I = H_0^I + H_1^I + H_p^I$ contains three parts

$$H_0^I = \Delta_1 S_z^{(1)} + \Delta_2 S_z^{(2)}, \tag{6}$$

$$H_1^I = \sum_{i=1,2} f_i \Omega (S_+^{(i)} B + S_-^{(i)} B^+), \tag{7}$$

$$H_p^I = e^{itH_0^S} H_p^S e^{-itH_0^S} \equiv H_p^S. \tag{8}$$

Here,

$$\Delta_j = \Omega_z - \omega_z + f_j I_0 \sum_i g_0^{(i)} (j = 1, 2)$$

are the detunings of electron spin and nuclear spin and hyperfine interaction.

It can be observed from the Hamiltonian H^I that the time-dependent term H_1^I can be considered as first-order perturbation with respect to the zeroth-order term H_0^I (disregarding H_p^I). Then, we perform a transformation $\exp(F(t))$ to the Hamiltonian H^I to eliminate the time-dependent term H_1^I, that is, the condition

$$H_1^I + [H_0^I, F] - i\partial_t F = 0 \tag{9}$$

should be fulfilled, where TDFT operator is

$$F(t) = (x_1(t) S_+^{(1)} + x_2(t) S_+^{(2)}) B - h.c.$$

It follows from equation (9) that the corresponding coefficients of $S_+^{(1)} B$, $S_-^{(1)} B^+$, $S_+^{(2)} B$ and $S_-^{(2)} B^+$ at the left hand side of equation (9) vanish, i.e.,

$$f_1 \Omega + \Delta_1 x_1 - i\dot{x}_1 = 0, \tag{10}$$

$$f_2 \Omega + \Delta_2 x_2 - i\dot{x}_2 = 0. \tag{11}$$

In case that the electrons go through the nuclear spins with a uniform speed v, the solutions to the above equations are $x_j \simeq -f_j\Omega/\Delta$ $(j=1,2)$, where $\Delta=\Omega_z-\omega_z$ is used to replace Δ_1 and Δ_2 since $\Delta\simeq\Delta_1\simeq\Delta_2$ in the realistic parameters (for the necessary details please refer to Appendix B). Then, the effective Hamiltonian is obtained approximately as follows:

$$\begin{aligned} H^F &\simeq H_0^I+\frac{1}{2}[H_1^I,F]+H_p^I+[H_p^I,F] \\ &\simeq \left[\Delta_1-\frac{f_1^2\Omega^2}{\Delta}(2\left\langle B^+B\right\rangle+1)\right]S_z^{(1)} \\ &\quad+\left[\Delta_2-\frac{f_2^2\Omega^2}{\Delta}(2\left\langle B^+B\right\rangle+1)\right]S_z^{(2)} \\ &\quad-\frac{f_1f_2\Omega^2}{\Delta}[S_+^{(1)}S_-^{(2)}+S_+^{(2)}S_-^{(1)}]+H_p^I+[H_p^I,F], \end{aligned} \tag{12}$$

where $\langle B^+B\rangle$ denotes the average number of nuclear excitation, and the fast-oscillating terms including the factor $\exp(\pm i\Delta t)$ have been dropped off.

When almost all nuclear spins are in their ground state, the system is in the low collective excitation limit, i.e., $\langle B^+B\rangle\to 0$. By using

$$|\Delta_{1,2}|\simeq|\Delta|\gg f_{1,2}\Omega,$$

we have

$$\Delta_j-\frac{f_j^2\Omega^2}{\Delta}(2\left\langle B^+B\right\rangle+1)\simeq\Delta_j\simeq\Delta.$$

Thus,

$$\begin{aligned} H^F &\simeq \Delta(S_z^{(1)}+S_z^{(2)})-\frac{f_1f_2\Omega^2}{\Delta}(S_+^{(1)}S_-^{(2)}+h.c.) \\ &\quad+H_p^I+[H_p^I,F]. \end{aligned} \tag{13}$$

In the following calculation, it will be shown that the complex term $[H_p^I,F]$ will be dropped in the interaction picture. With respect to $H_0=\Delta(S_z^{(1)}+S_z^{(2)})$, the effective interaction Hamiltonian is

$$H_{eff}=V_1+V_2, \tag{14}$$

where

$$\begin{aligned} V_1 &= -e^{iH_0t}\left[\frac{f_1f_2\Omega^2}{\Delta}(S_+^{(1)}S_-^{(2)}+S_+^{(2)}S_-^{(1)})\right]e^{-iH_0t} \\ &= -\frac{f_1f_2\Omega^2}{\Delta}(S_+^{(1)}S_-^{(2)}+S_+^{(2)}S_-^{(1)}), \end{aligned} \tag{15}$$

$$V_2=e^{itH_0}H_p^Ie^{-itH_0}=H_p^S. \tag{16}$$

In the above calculation, we have dropped the high-frequency term $\exp(iH_0t)[H_p^I,F]\exp(-iH_0t)$ including the factors $\exp(\pm i\Delta t)$. It is a reasonable approximation which is frequently used in the Jaynes-Cummings model.

Now, we study the time evolution driven by the above effective Hamiltonian. First of all, we study a special case that the total system is initially prepared without the collective excitations of the bus spins. In this case, the effective interaction V_2 does not play a role. In a Hilbert space spanned by the two electron states $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$, it is clear that there exists an invariant subspace spanned by $|eg\rangle$ and $|ge\rangle$. If the system starts from $|\Psi(0)\rangle=|eg\rangle$, at time t it would definitely evolve into

$$|\psi(t)\rangle=\cos\theta(t)\,|eg\rangle-i\sin\theta(t)\,|ge\rangle\,,$$

where

$$\theta(t)=-\int_0^t\frac{f_1f_2\Omega^2}{\Delta}dt'. \tag{17}$$

In comparison with the result in reference [12], where H_p was considered as a perturbation in the low excitation approximation, we examine the system evolving under total Hamiltonian containing V_1 and V_2. Then we can get the equations for the coefficients as follow

$$i\dot{C}_{ge}=-\frac{f_1f_2\Omega^2}{\Delta}C_{eg}-(V_2)_{eg}C_{ge}, \tag{18}$$

$$i\dot{C}_{eg}=(V_2)_{eg}C_{eg}-\frac{f_1f_2\Omega^2}{\Delta}C_{ge}, \tag{19}$$

where $(V_2)_{jk}=\langle jk|V_2\,|jk\rangle$ and $j,k=e,g$. Similarly, there's an invariant subspace $\{|eg\rangle\,,|ge\rangle\}$.

4 Spin entanglement

In the above sections, we have obtained a typical spin-spin coupling in the effective Hamiltonian, which is induced by the collective excitations. Driven by this Hamiltonian, two electron spins can be entangled dynamically. To characterize the extent of entanglement, we use concurrence to measure the induced entanglement. For an arbitrary state of two-qubit system described by the density operator ρ, a measure of entanglement can be defined as the concurrence [18,19],

$$C(\rho)=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}, \tag{20}$$

where the λ_i's are the square roots of the eigenvalues of the non-Hermitian matrix $\rho\widetilde{\rho}$ in decreasing order. And

$$\widetilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y),$$

where ρ^* is the complex conjugate of ρ, σ_y the Pauli operator. Actually, even from the original Hamiltonian (1) we can also prove that the corresponding reduced density matrix for two spins is of the form

$$\rho^{(12)}=\begin{pmatrix} u^+ & 0 & 0 & 0 \\ 0 & w^1 & z^* & 0 \\ 0 & z & w^2 & 0 \\ 0 & 0 & 0 & u^- \end{pmatrix}. \tag{21}$$

To prove the above result, we consider that, in the original Hamiltonian the interaction terms

$$H_I = S_z^{(1)} \sum_{j=1}^{N} g_1^{(j)} I_z^{(j)} + S_z^{(2)} \sum_{j=1}^{N} g_2^{(j)} I_z^{(j)} + S_+^{(1)} \sum_{l=1}^{N} \frac{g_1^{(j)}}{2} I_-^{(j)} + S_+^{(2)} \sum_{j=1}^{N} \frac{g_2^{(j)}}{2} I_-^{(j)} + h.c., \quad (22)$$

conserves the total spin z-component

$$\mathcal{S}_z = \sum_{j=1}^{N} I_z^{(j)} + S_z^{(1)} + S_z^{(2)},$$

i.e., $[H_I, \mathcal{S}_z] = 0$. For such conserved system we express the concurrence characterizing quantum entanglement in terms of observables, such as correlation functions.

The complete basis vectors of the total system are denoted by

$$|S_1, S_2, \{I_j\}\rangle = |S_1, S_2; I_1, .., I_N\rangle = \prod_{j=1}^{N} |I_j\rangle \otimes |S_1\rangle \otimes |S_2\rangle, \quad (23)$$

where $|I_j\rangle$ is nuclear spin state and $|S_{1,2}\rangle$ denote the electronic spins $(I_j, S_{1,2} = 0, 1)$ respectively. The fact that $\mathcal{S}_z$ is conserved can be reflected by the vanishing of some matrix elements of the density operator $\rho = \rho(H)$ on the above basis for any state of the total system, that is,

$$\rho_{S_1,S_2;I_1,..,I_N}^{S_1',S_2';I_1',..,I_N'} = \rho_{\{n_j,s_j\}}^{\{n_j,s_j\}} \delta(s, s', I, I'), \quad (24)$$

where

$$\delta(s, s', I, I') = \delta(S_1 + S_2 - S_1' - S_2' + \sum_{j=1}^{N} (I_j - I_j')).$$

The functional $\rho(H)$ of the Hamiltonian may be a ground state or thermal equilibrium states. The reduced density matrix $\rho^{(12)} = \mathrm{Tr}_I[\rho(H)]$ for two spins, e.g., S_1 and S_2 are obtained as

$$[\rho^{(12)}]_{S_1,S_2;}^{S_1',S_2';} = \delta(s, s', 0, 0) \sum_{\{I_j\}} \rho_{S_1,S_2;I_1,..,I_N}^{S_1',S_2';I_1,..,I_N} \quad (25)$$

by tracing over all nuclear variables. The corresponding reduced density matrix for two spins 1 and 2 is of the form in equation (21). Using the observable quantities, the quantum correlation

$$z = \langle\psi| S_1^+ S_2^- |\psi\rangle, \quad (26)$$
$$u^{\pm} = \langle\psi| (1/2 \pm S_1^z)(1/2 \pm S_2^z) |\psi\rangle,$$

the concurrence is rewritten as a computable form

$$C_{12} = 2 \max(0, |z| - \sqrt{u^+ u^-}). \quad (27)$$

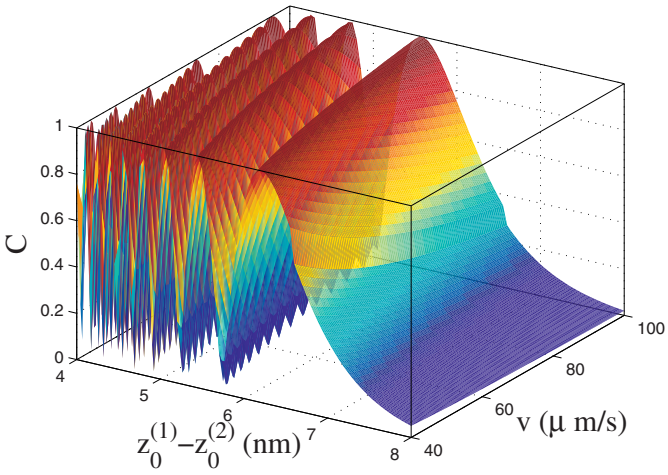

Fig. 2. (Color online) The relation between concurrence C and velocity v, distance $z_0^{(1)} - z_0^{(2)}$, on condition that $a = 4$ nm, $\Delta = 8 \times 10^{11}$ Hz, $I_0 = 3/2$, $n = 0$.

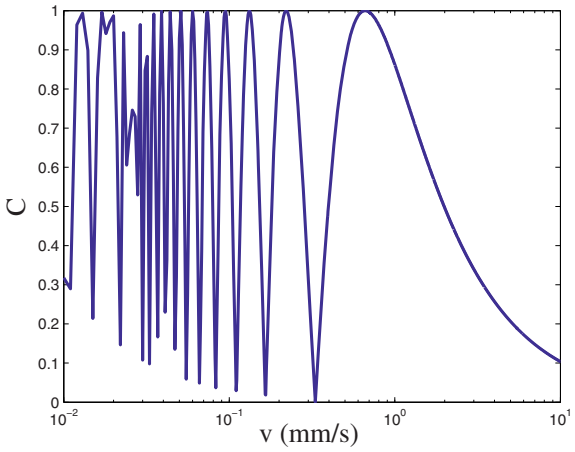

Fig. 3. Cross section of Figure 2 with $z_0^{(1)} - z_0^{(2)} = 4$ nm.

We note that this formula for the concurrence of two electron spins in the coupled system is the same as that for a spin-1/2 coupling system modeled by the effective Hamiltonian [18,19]. This general form is consistent with that obtained straightforwardly from the effective Hamiltonian given in the last section.

With the above general consideration, we now study quantitatively the concurrence for the quantum entanglement of the two electrons passing the nuclear spins at a uniform speed v. In Figure 2, the concurrence is plotted while the speed v and the initial distance between the two electrons $z_0^{(1)} - z_0^{(2)}$ are varied. It is obvious that the concurrence fluctuates from 0 to 1 in the low speed region (see also Fig. 3). When the electrons move with a relative low speed, the concurrence oscillates rapidly since a lower speed means more time for evolution from a direct product state towards an entangled state. As the speed increases, the concurrence falls monotonously if it is bigger than a certain value. According to equation (17), the maximum

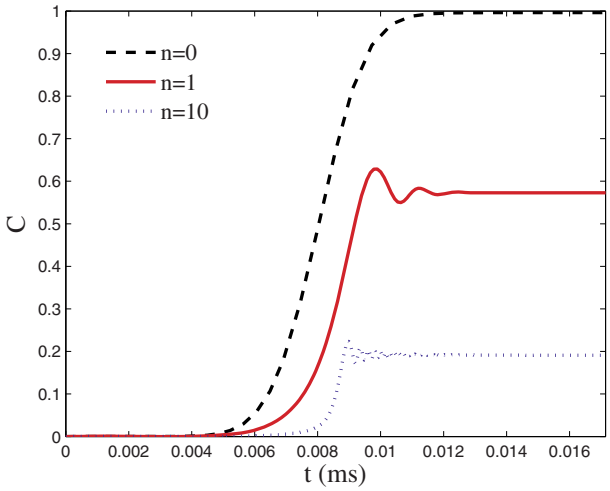

Fig. 4. (Color online) The evolution of concurrence C on condition that $a = 4$ nm, $\Delta = 8 \times 10^{11}$ Hz, $I_0 = 3/2$, $v = 0.7$ mm/s. Dashed line for $n = 0$, solid line for $n = 1$ and dotted line for $n = 10$, corresponding to nuclear spin polarization $P = 100\%$, $P = 99.993\%$, and $P = 99.93\%$, respectively.

entangled state can be obtained when

$$\theta = (n+1)\frac{\pi}{4}. \tag{28}$$

However, the general relation between the concurrence and $z_0^{(1)} - z_0^{(2)}$ is a little more complicated. The further the two electrons separate from each other, the longer time both of them need to pass through the nuclear ensemble. On the other hand, the matrix elements of the effective interaction V_1 in equation (15), i.e., $\langle\uparrow\downarrow |V_1| \downarrow\uparrow\rangle$, h.c., drop dramatically as the inter-spin distance increases. This observation is obviously correct from an intuitively physical consideration. For a longer inter-spin distance, the spatial wave functions of two spins have a smaller overlap, and then the effective coupling is weak.

In the last section, we have only taken V_1 into consideration. However, due to the nuclear excitation, i.e., $n > 0$, V_2 will lead to decoherence. According to reference [12], under the quasihomogeneous condition, we have $\sum_i g_0^{(i)}(I_z^{(i)} + I_0) = n\overline{g}$, where $\overline{g} = \sum_i g_0^{(i)}/N$, $n = \langle B^+ B\rangle$ is the average number of collective excitation. Thus, the single particle perturbation can be approximated as

$$V_2 = H_p^S \simeq n\overline{g}(f_1 S_z^{(1)} + f_2 S_z^{(2)}). \tag{29}$$

In Figure 4, we plot the concurrence evolution under $H_{eff} = V_1 + V_2$. As shown in the figure, the two spins evolve into a maximum entangled state with appropriate parameters when $n = 0$. On the contrary, the concurrence is suppressed when there exists collective excitation in the nuclei. Moreover, as more nuclei are excited, the concurrence falls dramatically. In recent experiments, for a typical quantum dot, the nuclear spin polarization P is of the order 60% [21]. And the relation between the number of collective excitation and nuclear spin polarization P is

$$P = 1 - \frac{n}{2NI_0}. \tag{30}$$

Thus, further progress in experiments, i.e., lowering the temperature or optical excitation, is expected to prepare all nuclear spins in their ground states in order to put this scheme into practice.

5 Conclusion

In summary, we have proposed a scheme to entangle two electron spins via an ensemble of nuclei. We also explore the influence of its collective excitation on the concurrence characterizing two spin entanglement. Theoretically, the maximum entangled state can be obtained if the electrons move in a suitable way. Furthermore, with the optimized experimental parameters, the operation time is within the relaxation time of electron spins in solid state systems, i.e., the order of ms [20].

However, this scheme may encounter some challenges from practical experiments since it is based on the low excitation requirement of nuclear ensemble. Moreover, there are only the collective excitations considered as the quantum data bus to coherently link two spins so that the inter-spin entanglement is induced. If there exist noncollective excitations, then extra decoherence will be induced to break our scheme presented in this paper. Further investigations are needed for these questions. However, if we can cool the nuclear ensemble via some new mechanism, e.g., similar to references [22,26], our scheme will probably work well.

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Appendix A: Derivation of $g_1^{(i)}$ and $g_2^{(i)}$

According to reference [23], the hyperfine interaction constant is mainly proportional to the electron spin density located at the nucleus. Thus,

$$g_1^{(i)} = \frac{4\mu_0}{3I_0\hbar}\mu_B\mu_I \left|\psi_1^{(i)}\right|^2, \tag{31}$$

where μ_0 is vacuum permeability, I_0 total nuclear spin quantum number, μ_B the Bohr magneton, μ_I the nuclear magnetic moment, $\psi_1^{(i)}$ the wavefunction for electron 1 located at the ith nuclear spin. In a semiconductor crystal, the wavefunction is given by the product of the Bloch amplitude $u(\boldsymbol{r})$ and an envelope function $\Psi(\boldsymbol{r})$, i.e., $\psi(\boldsymbol{r}) = u(\boldsymbol{r})\Psi(\boldsymbol{r})$. In a realistic crystal $\eta = |u(\boldsymbol{r})|^2$ reaches a climax at the lattice positions. It was discovered that $\eta_{\rm As} = 4.5 \times 10^3$ and $\eta_{\rm Ga} = 2.7 \times 10^3$ [24]. On account of isotope abundance and their different nuclear magnetic moments [25], $\overline{\mu_I \eta} = 3.153 \times 10^{-23}$ A.m^2. In our simulation, we assume that the wavefunction is a Gaussian wave

packet with initial location $z_0^{(1)}$ and moves at a uniform speed v, that is

$$\left|\psi_1^{(i)}\right|^2 = \left(\frac{4}{\pi a^2}\right)^{3/2} e^{-4[x_i^2+y_i^2+(z_0^{(1)}+vt-z_i)^2]/a^2}, \quad (32)$$

where $\boldsymbol{r}_i = x_i\hat{x}+y_i\hat{y}+z_i\hat{z}$ is the position of the ith nuclear spin with respect to the center of nuclei.

Since the nuclei are distributed in a flat box with $|x_i| \leq a$, $|y_i| \leq a$, $|z_i| \leq a/5$. Therefore,

$$g_1^{(i)} \simeq \frac{4\mu_0}{3I_0\hbar}\mu_B\mu_I\eta\left(\frac{4}{\pi a^2}\right)^{3/2} e^{-4r_i^2/a^2}e^{-4(z_0^{(1)}+vt)^2/a^2} = g_0^{(i)}f_1(t), \quad (33)$$

where

$$g_0^{(i)} = \frac{4\mu_0}{3I_0\hbar}\mu_B\mu_I\eta\left(\frac{4}{\pi a^2}\right)^{3/2} e^{-4r_i^2/a^2}, \quad (34)$$

$$f_1(t) = e^{-4(z_0^{(1)}+vt)^2/a^2}. \quad (35)$$

Here, we have neglected the term $e^{-8(z_0^{(1)}+vt)z_i/a^2}$ based on the following consideration. On the one hand, the above approximation tends to be exact as the quasi-2D quantum well becomes narrower in the z-direction, e.g., $|z_i| \to 0$. On the other hand, the effective coupling intensity is increased as more nuclear spins are included when $|z_i|$ gets larger. Thus, optimal value is chosen for the valid approximation. Similarly, we have

$$g_2^{(i)} \simeq g_0^{(i)}f_2(t), \quad (36)$$

where

$$f_2(t) = e^{-4(z_0^{(2)}+vt)^2/a^2}. \quad (37)$$

Appendix B: Derivation of x_i

According to equation (10), one has

$$\begin{aligned} x_1 &= -i\Omega e^{-i\int_0^t \Delta_1 dt'}\int_0^t f_1 e^{i\int_0^{t'}\Delta_1 dt''}dt' \\ &\simeq -i\Omega e^{-i\Delta t}\int_{-\infty}^t f_1 e^{i\Delta t'}dt' \\ &= \frac{-\Omega}{\Delta}e^{-i\Delta t}[f_1 e^{i\Delta t} - \frac{8v}{a^2}\int_{-\infty}^t f_1 e^{i\Delta t'}(z_0^{(1)}+vt')dt'] \\ &\simeq \frac{-\Omega}{\Delta}e^{-i\Delta t}(f_1 e^{i\Delta t} - \frac{8v}{a}\int_{-\infty}^t f_1 e^{i\Delta t'}dt'). \end{aligned}$$

Here, we have replaced Δ_1 by $\Delta = \Omega_z - \omega_z$ since

$$f_1 I_0 \sum_i g_0^{(i)} \lesssim 8.318\times 10^{10}\ \text{Hz} \ll \Delta = 8\times 10^{11}\ \text{Hz}.$$

Furthermore, we have replaced

$$\frac{8v}{a^2}\int_{-\infty}^t f_1 e^{i\Delta t'}[z_0^{(1)}+vt']dt'$$

by

$$\frac{8av}{a^2}\int_{-\infty}^t f_1 e^{i\Delta t'}dt',$$

since one notices the fact that $z_0^{(1)}+vt' \sim a$ is the effective integration range and the change of $f_1(t')$ (also change of vt') is much slower than that of $e^{i\Delta t'}$. Thus,

$$x_1 \simeq \frac{-\Omega}{\Delta}f_1 - i\frac{8v}{\Delta a}x_1.$$

Generally speaking, Δ depends on the applied magnetic field. In case that $\Delta = 8\times 10^{11}$ Hz and $a \sim 4$ nm, we have $8v/a\Delta \ll 1$ for all $v \ll 400$ m/s. Thus, we have

$$x_1 \simeq \frac{-\Omega}{\Delta}f_1. \quad (38)$$

Similarly,

$$x_2 \simeq \frac{-\Omega}{\Delta}f_2. \quad (39)$$

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