Impulsive synchronization of networked nonlinear dynamical systems

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Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Multi-agent systems

- Recently, multi-agent systems have been intensively studied in various disciplines.
    - The goal of multi-agent systems is to generate a desired collective behavior by local interaction among the agents, such as group consensus, group coordination, oscillator synchronization and so on.
    - In the real word the communication topologies of the multi-agent systems are dynamically changing over time. Very recently some consensus, synchronization and coordination problems of multi-agent systems have received much attention.

Impulsive control protocol

- Impulsive control is widely used in various applications, such as ecosystems, financial systems, mechanical systems with impacts, orbital transfer of satellite.
    - Recently, the problem for impulsive synchronization of chaotic systems and complex networks has sparked the interest of many researchers.
    - However, impulsive control protocol for multi-agent systems has received relatively little attention.

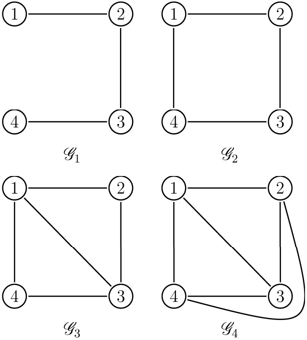
Impulsive control protocol

- In [1] Several criteria related to the eigenvalues and eigenvectors of coupling matrix for synchronizing a kind of impulsively coupled complex dynamical systems were established.
  [1] X.P. Han, J.A. Lu, X.Q. Wu, Synchronization of impulsively coupled systems, Int. J. Bifur. Chaos 18 (2008) 1539-1549.
- In [2], the authors investigated the problem of average consensus in delayed networks of dynamic agents with impulsive effects.
  [2] Q.J. Wu, L. Xiang, J. Zhou, Average consensus in delayed networks of dynamic agents with impulsive effects, in: J. Zhou (Eds.), Complex Sciences, Springer, Berlin, 2009, pp. 1124-1138.

Impulsive control protocol

- In [3], the authors introduced impulsive control protocols for multi-agent linear continuous dynamic systems. The convergence analysis of the impulsive control protocol for networks with fixed and switching topologies is presented, respectively.
  [3] H.B. Jiang, J.J. Yu, C.G. Zhou, Consensus of multi-agent linear dynamic systems via impulsive control protocols, Int. J. Systems Sci., 2010, doi:10.1080/00207720903267866.
- In [4], the authors studied synchronization problems of complex dynamical networks (CDNs) via distributed impulsive control.
  [4] Z.H. Guan, Z.W. Liu, G. Feng, Y.W. Wang, Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control, IEEE Trans. Circuits Syst.-I, doi:10.1109/TCSI.2009.2037848.

Example–Undirected graph



Preliminaries–Undirected graph

A weighted adjacency matrix $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0$, $i \neq j$. $a_{ij} > 0$ if and only if there is an edge between vertex i and vertex j. For an unweighted graph $\mathscr{G}$, $\mathscr{A}$ is a 0-1 matrix. The out-degree of vertex i is defined as follows $\deg_{\text{out}}(i) = \sum_{j=1}^{n} a_{ij}$. Let $\mathscr{D}$ be the diagonal matrix with the out-degree of each vertex along the diagonal and call it the degree matrix of $\mathscr{G}$. The Laplacian matrix of the weighted graph is defined as $L_{\mathscr{G}} = \mathscr{D} - \mathscr{A}$. For an unweighted graph $\mathscr{G}$,

$$L_{\mathscr{G}} = [l_{ij}]_{N \times N}, \tag{2.1}$$

where

$$l_{ij} = \begin{cases} |\mathscr{N}_i|, \ i = j, \\ -1, \ j \in \mathscr{N}_i, \\ 0, \text{otherwise}. \end{cases} \tag{2.2}$$

Lemma–Undirected graph

Lemma (1)

Let L be the Laplacian of an undirected graph $\mathscr{G}$ with N vertices, $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ be the eigenvalues of L. Let $\mathbf{1}_N = (1, 1, \cdots, 1)^T \in \mathbb{R}^N$ and $e_i \in \mathbb{R}^n$, $e_i(i) = 1$, $e_i(j) = 0$, $j \neq i$. Then

(1) 0 is an eigenvalue of L and $\mathbf{1}_N$ is the associated eigenvector, that is, $L\mathbf{1}_N = 0$;

(2) If $\mathscr{G}$ is connected, then $\lambda_1 = 0$ is the algebraically simple eigenvalue of L.

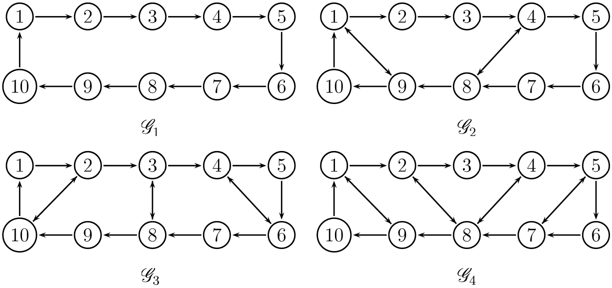
(3) If 0 is the simple eigenvalue of L, then it is an n multiplicity eigenvalue of $L \otimes I_n$ and the corresponding eigenvectors are $\mathbf{1}_N \otimes e_i$, $i = 1, 2, \cdots, n$.

Lemma–Undirected graph

Lemma (2)

Let L be the Laplacian of an undirected connected graph $\mathscr{G}$ with N vertices, $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$ be the eigenvalues of L. Then
(1) The eigenvalues of LL are $0 = (\lambda_1)^2 < (\lambda_2)^2 \leq \cdots \leq (\lambda_N)^2$.
(2) The eigenvalues of LLL are $0 = (\lambda_1)^3 < (\lambda_2)^3 \leq \cdots \leq (\lambda_N)^3$.
(3) Let $c_1 = (\lambda_2)^2/\lambda_N$, $c_2 = (\lambda_N)^3/\lambda_2$, then $LL \geq c_1 L$ and $LLL \leq c_2 L$.

Example–Directed graph



Preliminaries–Directed graph

The graph is said to be balanced if and only if every vertex's in-degree and out-degree are equal, i.e. $\sum_{j=1}^{n} a_{ji} = \sum_{j=1}^{n} a_{ij}$, $i = 1, 2, \cdots, N$. If the graph is balanced, then $\mathbf{1}^T L = 0$. Given $C = [c_{ij}] \in R^{N\times r}$, it is said that $C \geq 0$ (C is nonnegative) if all its elements c_{ij} are nonnegative, and it is said that $C > 0$ (C is positive) if all its elements c_{ij} are positive. Further, $C \geq D$ if $C - D \geq 0$, and $C > D$ if $C - D > 0$. If a nonnegative matrix $C \in R^{n\times n}$ satisfies $C\mathbf{1} = \mathbf{1}$, then it is said to be stochastic. A square matrix $C \in R^{N\times N}$ is said to be doubly stochastic if both C and C^T are stochastic.

Lemma–Directed graph

Let L be the graph Laplacian of the network. We refer to $P = I - \varepsilon L$ as Perron matrix of a graph $\mathscr{G}$ with parameter ε.

Lemma (3)

Let $\mathscr{G}$ be a directed graph with n nodes and maximum degree $d = \max_i(\sum_{j \neq i} a_{ij})$. Then, the perron matrix P with parameter $\varepsilon \in (0, 1/d]$ satisfies the following properties.
(1) P is a row stochastic nonnegative matrix with a trivial eigenvalue of 1;
(2) All eigenvalues of P are in a unit circle;
(3) If $\mathscr{G}$ is a balanced graph, then P is a doubly stochastic matrix.

Dynamics of MAS

Here we consider a system consisting of N agents indexed by $i = 1, 2, \ldots, N$. The dynamics of each agent is

$$\dot{x}^i(t) = f(x^i(t), t) + u^i(t), x^i(t) \in \mathbb{R}^n, t \geq t_0 \geq 0, i = 1, 2, \ldots, N, \tag{3.1}$$

where $x^i(t) = (x^i_1(t), x^i_2(t), \ldots, x^i_n(t))^T \in \mathbb{R}^n$ and $u^i(t) \in \mathbb{R}^n$ are the state and the control input of agent i at time t, respectively; $f(x^i(t), t) \in \mathbb{R}^n$ is the nonlinear vector field function of agent i at time t.

Impulsive control protocol

The control input of agent i is designed as

$$u^i(t) = \sum_{k=1}^{+\infty} \delta(t - t_k) B_k \sum_{j \in \mathcal{N}_i(t)} (x^j(t) - x^i(t)), k \in \mathbb{N}_+, i = 1, 2, \ldots, N, \tag{3.2}$$

where the discrete instants t_k satisfy $0 \leq t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, and $\lim_{k \to +\infty} t_k = +\infty$, $\delta(t)$ is the Dirac delta function, i.e., $\delta(t) = 0$ for $t \neq 0$, and $\int_{-\infty}^{\infty} \delta(t) \mathrm{d}t = 1$. $B_k \in \mathbb{R}^{n \times n}$, $k \in \mathbb{N}_+$ are impulsive matrices to be designed later, $\mathcal{N}_i(t)$ is the set of neighbors of agent i at time t. Without loss of generality, we assume that $\lim_{t \to t_k^+} x^i(t) = x^i(t_k)$, which means that the solution $x^i(t)$ is right continuous at time t_k.

Impulsive control protocol

From (3.1) and (3.2) we have

$$x^i(t_k+\varepsilon)-x^i(t_k-\varepsilon)=\int_{t_k-\varepsilon}^{t_k+\varepsilon}(f(x^i(s),s)+u^i(s))\mathrm{d}s,$$

where $\varepsilon > 0$ is sufficiently small. As $\varepsilon \to 0^+$, this becomes to $\Delta x^i(t_k) = B_k \sum_{j\in\mathcal{N}_i}(x^j(t_k^-)-x^i(t_k^-))$, where $\Delta x^i(t_k) = x^i(t_k^+)-x^i(t_k^-)$, $x^i(t_k^+) = \lim_{t\to t_k^+} x^i(t)$ and $x^i(t_k^-) = \lim_{t\to t_k^-} x^i(t)$. This implies that the agent i will suddenly update its state variable according to the state variables of itself and its neighbors at the instants t_k. Thus the control input $u^i(t)$ is called an impulsive control protocol.

Main problem

For simplicity, in the following we choose $B_k = b_k I_n$, $k \in \mathbb{N}_+$. Then under the impulsive control protocol (3.2), the dynamics of agent i satisfies the following equations

$$\begin{cases} \dot{x}^i(t) = f(x^i(t), t), t \neq t_k, \\ \Delta x^i(t_k) = x^i(t_k^+) - x^i(t_k^-) = b_k \sum\limits_{j \in \mathcal{N}_i(t_k^-)} (x^j(t_k^-) - x^i(t_k^-)), i = 1, 2, .. \end{cases} \tag{3.3}$$

Definition (1)

For system (3.1), the agents are said to be synchronized under the impulsive control protocol (3.2) if

$$\lim_{t \to +\infty} ||e^{i,j}(t)|| = 0, i, j = 1, 2, \ldots, N, \tag{3.4}$$

where $e^{i,j}(t) = x^i(t) - x^j(t)$.

Assumption

Assumption (1)

For any $x(t), y(t) \in \Omega \subseteq \mathbb{R}^n$, *there exists a constant* $\theta = \theta(\Omega)$, *such that*

$$(x(t)-y(t))^T(f(x(t),t)-f(y(t),t)) \leq \theta(x(t)-y(t))^T(x(t)-y(t)),$$

where Ω *is a bounded set.*

Network With Fixed Topology

In this section, we provide the analysis of the impulsive synchronization problem for network with fixed topology, i.e. $\mathscr{G}(t) = \mathscr{G}$ for time t.
Let $x(t) = (x^1(t), x^2(t), \ldots, x^N(t))^T$, then system (3.3) can be described as

$$\begin{cases} \dot{x}(t) = F(x(t), t), t \neq t_k, \\ \Delta x(t_k) = (-b_k L \otimes I_n) x(t_k^-), k \in \mathbb{N}_+. \end{cases} \tag{4.1}$$

where $F(x(t), t) = (f(x^1(t), t), f(x^2(t), t), \cdots, f(x^N(t), t))^T$.
Then, we get

$$\begin{cases} \dot{x}(t) = F(x(t), t), t \neq t_k, \\ x(t_k^+) = ((I_N - b_k L) \otimes I_n) x(t_k^-), k \in \mathbb{N}_+. \end{cases} \tag{4.2}$$

Main results–Theorem 1

Theorem (1)

Consider system (3.1) with Assumption 1. Assume that the graph $\mathscr{G}$ of the network is connected. If there exist discrete instants t_k and impulsive constants b_k such that the conditions (i) and (ii) hold, then the agents are synchronized under the impulsive control protocol (3.2).

(i) There exist two constants β_1 and β_2 such that $0 < \beta_1 \leq t_k - t_{k-1} \leq \beta_2 < +\infty$, $k \in \mathbb{N}_+$;

(ii) There exist some constants $0 < \alpha_k < 1$ and $0 < \gamma < 1$ such that $(1 - \alpha_k)L - 2b_k LL + (b_k)^2 LLL \leq 0$, and $\alpha_k e^{2\theta(t_k - t_{k-1})} \leq \gamma < 1$, $k \in \mathbb{N}_+$.

Proof of Theorem 1

Proof.

Consider the Lyapunov function candidate

$$V(x(t)) = \sum_{i=1}^{N} V_i(x(t)) = \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} (x^j(t) - x^i(t))^T (x^j(t) - x^i(t))/2$$
$$= x^T(t)(L \otimes I_n)x(t)/2.$$

Taking the Dini derivative of $V(x(t))$ for $t \in [t_{k-1}, t_k)$, $k \in \mathbb{N}_+$, by Assumption 1, we obtain $D^+V(x(t)) \leq 2\theta V(x(t))$. Then

$$V(x(t)) \leq e^{2\theta(t-t_{k-1})} V(x(t_{k-1}^+)), t \in [t_{k-1}, t_k), k \in \mathbb{N}_+. \qquad (4.3)$$

□

Network With Fixed Topology

Proof.

On the other hand, when $k \in \mathbb{N}_+$, by condition (ii) we have

$$V(x(t_k^+)) \leq \alpha_k V(x(t_k^-)).$$

By mathematical induction, one can easily show that

$$V(x(t)) \leq e^{2\theta(t-t_{k-1})} \Pi_{j=1}^{k-1} \alpha_j e^{2\theta(t_j - t_{j-1})} V(x(t_0^+)), t \in [t_{k-1}, t_k), k \in \mathbb{N}_+, \tag{4.4}$$

From conditions (i) and (ii), we get

$$V(x(t)) \leq e^{2|\theta|\beta_2} \gamma^k V(x(t_0^+)), t \in [t_{k-1}, t_k), k \in \mathbb{N}_+, k \geq 2.$$

Thus $V(x(t)) \to 0$ as $t \to \infty$. Since the graph $\mathscr{G}$ of the network is connected, it follows that $||x^i(t) - x^j(t)|| \to 0$ as $t \to \infty$, $i, j = 1, 2, \ldots, N$. □

Main results–Corollary 1

Corollary (1)

Consider system (3.1) with Assumption 1. Assume that the graph $\mathscr{G}$ of the network is connected. Choose $b_k = p$, $k \in \mathbb{N}_+$, and $0 < p < c_1/c_2$, where $c_1 = (\lambda_2)^2/\lambda_N$, $c_2 = (\lambda_N)^3/\lambda_2$. Choose $\alpha_k = q > 0$, $k \in \mathbb{N}_+$, and $1 - c_1 p \leq q < 1$. If we choose the equidistant impulsive interval $\Delta t_k = t_k - t_{k-1} = \Delta$, $k \in \mathbb{N}_+$, such that

$$0 < \Delta < \frac{\ln \frac{1}{q}}{2\theta},$$

then the agents are synchronized under the impulsive control protocol (3.2).

Networks With Switching Topologies

In this section, we provide the analysis of the impulsive synchronization problem for networks with switching topologies. Here we consider m graphs indexed by $\mathscr{G}_1, \mathscr{G}_2, \cdots, \mathscr{G}_m$. We define a switching signal $\sigma : [t_0, +\infty) \to \{1, 2, \cdots, m\}$. The switching signal is a piecewise constant right continuous function. Suppose that $\sigma(t_k^-) = \tau_k^-$ and its graph is $\mathscr{G}_{\tau_k^-}$ with the Laplacian $L_{\tau_k^-}$, where $\tau_k^- \in \{1, 2, \cdots, m\}$, $k \in \mathbb{N}_+$.

Networks With Switching Topologies

Under the impulsive control protocol (3.2), the dynamics of agent i satisfies the following equations

$$\begin{cases} \dot{x}^i(t) = f(x^i(t), t), t \neq t_k, \\ \Delta x^i(t_k) = x^i(t_k^+) - x^i(t_k^-) = b_k \sum\limits_{j \in \mathcal{N}_i(t_k^-)} (x^j(t_k^-) - x^i(t_k^-)), i = 1, 2, \ldots \end{cases} \tag{4.5}$$

Let $x(t) = (x^1(t), x^2(t), \ldots, x^N(t))^T$, then system (4.5) can be described as

$$\begin{cases} \dot{x}(t) = F(x(t), t), t \neq t_k, \\ \Delta x(t_k) = (-b_k L_{\tau_k^-} \otimes I_n) x(t_k^-), k \in \mathbb{N}_+. \end{cases} \tag{4.6}$$

Then we get

$$\begin{cases} \dot{x}(t) = F(x(t), t), t \neq t_k, \\ x(t_k^+) = ((I_N - b_k L_{\tau_k^-}) \otimes I_n) x(t_k^-), k \in \mathbb{N}_+. \end{cases} \tag{4.7}$$

Main results–Theorem 2

Theorem (2)

Consider system (3.1) with Assumption 1. Assume that the networks are switching and the graphs $\mathscr{G}_i$, $i = 1, 2, \cdots, m$, are connected. If there exist discrete instants t_k and impulsive constants b_k such that the conditions (i) and (ii) hold, then the agents are synchronized under the impulsive control protocol (3.2).
(i) There exist two constants β_1 and β_2 such that
$0 < \beta_1 \leq t_k - t_{k-1} \leq \beta_2 < +\infty$, $k \in \mathbb{N}_+$;
(ii) There exist some constants $0 < \alpha_k < 1$ and $0 < \gamma < 1$ such that

$$L_{\tau_{k+1}^-} - 2b_k L_{\tau_{k+1}^-} L_{\tau_{k+1}^-} + (b_k)^2 L_{\tau_{k+1}^-} L_{\tau_{k+1}^-} L_{\tau_{k+1}^-} - \alpha_k L_{\tau_k^-} \leq 0,$$

and $\alpha_k e^{2\theta(t_k - t_{k-1})} \leq \gamma < 1$, $k \in \mathbb{N}_+$.

Case II: Directed networks

Assumption (2)

For any $x(t), y(t) \in \Omega \subseteq \mathbb{R}^n$, *there exists a constant* $\theta = \theta(\Omega)$, *such that* $||f(x(t),t) - f(y(t),t))|| \leq \theta||x(t) - y(t)||$, *where* Ω *is a bounded set.*

Assumption (3)

The graphs $\mathscr{G}_i$, $i = 1, 2, \cdots, m$ *of the networks are strongly connected and balanced.*

Let $x(t) = (x^1(t), x^2(t), \ldots, x^N(t))^T$, then the system (3.3) can be described as

$$\begin{cases} \dot{x}(t) = F(x(t), t), t \neq t_k, \\ x(t_k^+) = (P_{t_k^-} \otimes I_n)x(t_k^-), k \in \mathbb{N}_+. \end{cases} \tag{5.1}$$

where $P_{t_k^-} = I_N - b_k L_{\tau_k^-}$.

Case II: Directed networks

Define

$$\bar{x}(t) = \frac{1}{N}\sum_{i=1}^{N} x^i(t) = \frac{1}{N}(\mathbf{1}^T \otimes I_n)x(t),$$

then by Assumption 2 and Lemma 2 we have

$$\begin{aligned}\bar{x}(t_k^+) &= \frac{1}{N}(\mathbf{1}^T \otimes I_n)(P_{t_k^-} \otimes I_n)x(t_k^-) \\ &= \frac{1}{N}(\mathbf{1}^T P_{t_k^-} \otimes I_n)x(t_k^-) \\ &= \bar{x}(t_k^-).\end{aligned}$$

Therefore the dynamics of $\bar{x}$ satisfies the following equations

$$\begin{cases} \dot{\bar{x}}(t) = \frac{1}{N}\sum_{i=1}^{N} f(x^i(t), t), t \neq t_k, \\ \bar{x}(t_k^+) = x(t_k^-), k \in \mathbb{N}_+. \end{cases} \quad (5.2)$$

Main results–Theorem 3

Theorem (3)

Consider system (3.1) with Assumptions 2 and 3. If there exist discrete instants t_k *and impulsive constants* b_k *such that the conditions (i)-(iii) hold, then the consensus is said to be achieved under the impulsive control protocol (3.2).*

(i) There exist two constants β_1 *and* β_2 *such that*
$0 < \beta_1 \leq t_k - t_{k-1} \leq \beta_2 < +\infty,\ k \in \mathbb{N}_+;$

(ii) There exists some constants $b_k > 0$, $\delta_k > 0$ *such that* $P_{t_k^-} = I_N - b_k L_{\tau_k^-}$ *are nonnegative matrices with positive diagonal entries and every nonzero entry of* $P_{t_k^-}$ *is no smaller than* δ_k*;*

(iii) There exists a constant $\mu > 0$ *such that*
$(1 - \delta_k^2/N)e^{4\theta(t_k - t_{k-1})} \leq \mu < 1,\ k \in \mathbb{N}_+.$

Example 1

Consider the following networked nonlinear dynamical system, which consists of two duffing systems,

$$\dot{x}^i(t) = f(x^i(t), t) + u^i(t), i = 1, 2, \tag{6.1}$$

where $x^i = (x_1^i,\ x_2^i)^T$,
$f(x^i(t), t) = (x_2^i(t),\ x_1^i(t) - (x_1^i(t))^3 - \delta x_2^i(t) + \gamma \cos(\omega t))^T$,
$\delta = 0.25$, $\gamma = 0.4$, $\omega = 1$. The control input of agent i is designed as

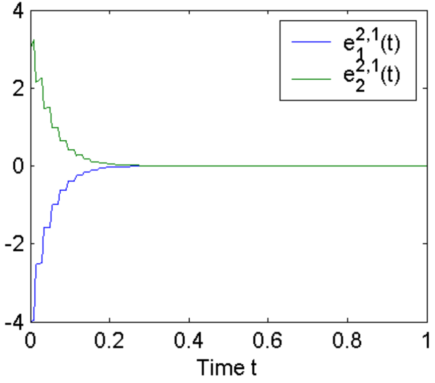
$$\begin{aligned} u^1(t) &= \sum_{k=1}^{+\infty} \delta(t - t_k) b_k (x^2(t) - x^1(t)), k \in \mathbb{N}_+, \\ u^2(t) &= \sum_{k=1}^{+\infty} \delta(t - t_k) b_k (x^1(t) - x^2(t)), k \in \mathbb{N}_+. \end{aligned} \tag{6.2}$$

Example 1

By some computation we get $\lambda_2 = 2$, $c_1 = (\lambda_2)^2/\lambda_2 = 2$, $c_2 = (\lambda_2)^3/\lambda_2 = 4$, $\theta = 11$. For simplicity, choose the impulsive constants $0 < b_k = p = 0.2 < c_1/c_2$, $\alpha_k = q = 0.6$, $k \in \mathbb{N}_+$, and the equidistant impulsive interval $\Delta t_k = t_k - t_{k-1} = \Delta = 0.02 < \ln(1/q)/(2\theta) = 0.0232$. From Corollary 1, we know that the synchronization is achieved. The initial values are chosen as $x^1(0) = (3\ \ -1)^T$, $x^2(0) = (-1\ 2)^T$. Simulation results are shown in Figs. 1-2.

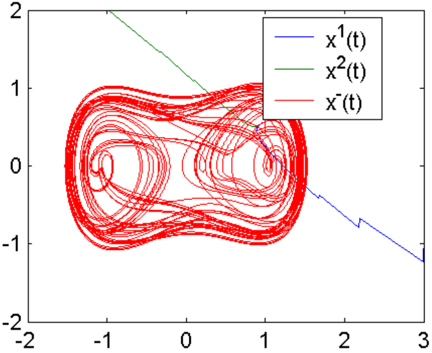
Simulation results–Example 1

Fig.1 The time histories of $e^{2,1}(t)$



Simulation results–Example 1

Fig.2 The phase graph of $x^1(t)$, $x^2(t)$ and $\bar{x}(t)$



Example 2

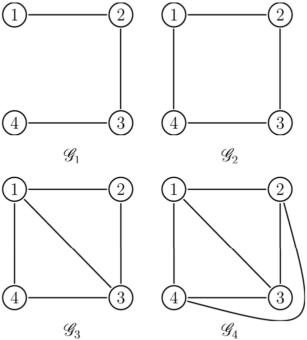
The chaotic Chua's circuit is used as agent of the networked nonlinear dynamical system. The state equation of agent i is

$$\begin{cases} \dot{x}_1^i(t) = \eta(-x_1^i(t) + x_2^i(t) - l(x_1^i(t))), \\ \dot{x}_2^i(t) = x_1(t) - x_2^i(t) + x_3^i(t), \\ \dot{x}_3^i(t) = -\beta x_2^i(t), \end{cases} \tag{6.3}$$

where $l(x_1^i(t)) = bx_1^i(t) + 0.5(a-b)(|x_1^i(t)+1| - |x_1^i(t)-1|)$. When $\eta = 10$, $\beta = 18$, $a = -4/3$, and $b = -3/4$, system (6.3) is chaotic.

Example 2

Fig.3 Schematic representation of $\mathscr{G}_1$, $\mathscr{G}_2$, $\mathscr{G}_3$ and $\mathscr{G}_4$

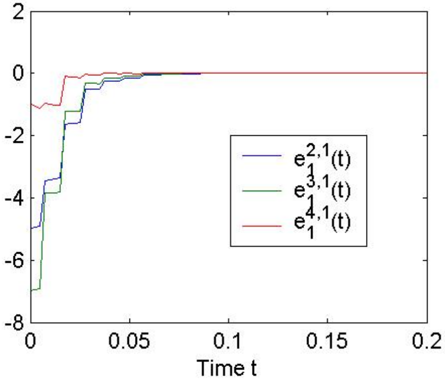


Example 2

In the following, we give simulation results of the synchronization problem for networks with switching topologies. Here we consider 4 graphs indexed by $\mathscr{G}_1, \mathscr{G}_2, \mathscr{G}_3, \mathscr{G}_4$. We define a switching signal $\sigma : [t_0, +\infty) \to \{1, 2, 3, 4\}, \sigma(t) = 4 - ((k-1) \bmod 4)$, $t \in [t_{k-1}, t_k)$. For simplicity, choose the impulsive constants $b_k = p = 0.25$, $k \in \mathbb{N}_+$, and the equidistant impulsive interval $\Delta t_k = t_k - t_{k-1} = \Delta = 0.01$, $\alpha_k = 0.9$, $k \in \mathbb{N}_+$. It is easy to check that $\alpha_k e^{2\theta(t_k - t_{k-1})} = \gamma = 0.9979 < 1$, where $\theta = 5.1623$. Thus the conditions (i) and (ii) in Theorem 2 are satisfied. Simulation results are shown in Figs. 4-6.

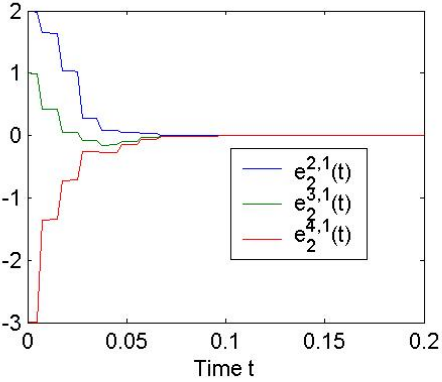
Simulation results–Example 2

Fig.4 The time histories of $e_1^{i,1}(t)$, $i = 2, 3, 4$



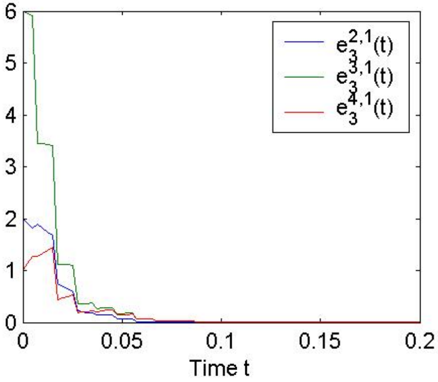
Simulation results–Example 2

Fig.5 The time histories of $e_2^{i,1}(t)$, $i = 2, 3, 4$



Simulation results–Example 2

Fig.6 The time histories of $e_3^{i,1}(t)$, $i = 2, 3, 4$



Example 3

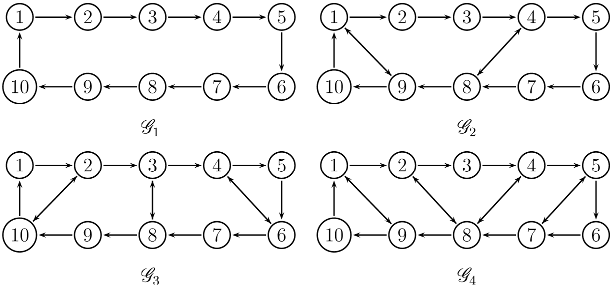
The chaotic Chua's circuit is used as agent of the networked nonlinear dynamical system. The state equation of agent i is

$$\begin{cases} \dot{x}_1^i(t) = \eta(-x_1^i(t) + x_2^i(t) - l(x_1^i(t))) + u_1^i(t), \\ \dot{x}_2^i(t) = x_1(t) - x_2^i(t) + x_3^i(t) + u_2^i(t), \\ \dot{x}_3^i(t) = -\beta x_2^i(t) + u_3^i(t), \end{cases} \tag{6.4}$$

where $l(x_1^i(t)) = bx_1^i(t) + 0.5(a-b)(|x_1^i(t)+1| - |x_1^i(t)-1|)$, $i = 1, 2, \cdots, 10$. When $\eta = 10$, $\beta = 18$, $a = -4/3$, $b = -3/4$ and $u^i(t) = 0$, system (6.3) is chaotic. The graphs $\mathscr{G}_i$, $i = 1, 2, 3, 4$, are shown in Fig. 7.

Example 3

Fig.7 Schematic representation of $\mathscr{G}_1$, $\mathscr{G}_2$, $\mathscr{G}_3$ and $\mathscr{G}_4$

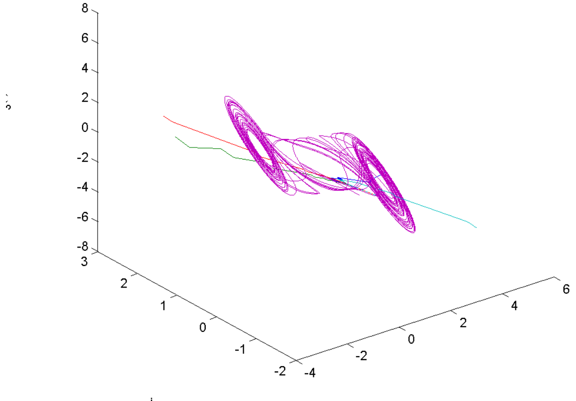


Example 3

We define a switching signal $\sigma : [t_0, +\infty) \to \{1, 2, 3, 4\}, \sigma(t) = ((k-1) \bmod 4) + 1$, $t \in [t_{k-1}, t_k)$. For simplicity, choose the impulsive constants $b_k = p = 0.2$, $k \in \mathbb{N}_+$, and the equidistant impulsive interval $\Delta t_k = t_k - t_{k-1} = \gamma = 0.00003$, $k \in \mathbb{N}_+$. It is easy to check that $(1-(\delta_k)^2/N)e^{4\theta(t_k - t_{k-1})} \leq (1-(\bar{\delta})^2/N)e^{4\theta\gamma} = \mu = 0.9998 < 1$, where $\bar{\delta} = 0.2$, $\theta = 31.7763$. Thus the conditions (i)- (iii) in Theorem 3 are satisfied. The initial values are randomly chosen in the interval $[-5, 5]$. Simulation results are shown in Figs.8. The simulation results show that the impulsive control protocol is efficient to solve the consensus problem for the networks with switching topologies.

Simulation results–Example 3

Fig.8 The phase graph of $x^i(t)$, $i = 1, 2, \cdots, 10$ and $\bar{x}(t)$



Conclusions

- We have investigated the problem of impulsive synchronization of networked nonlinear dynamical systems.
  - Case I: Undirected networks. A design scheme of the discrete instants and impulsive constants is given for network with fixed topology by the largest and the second smallest eigenvalues of the Laplacian matrix and a design procedure is given for networks with switching topologies.
  - Case II: Directed networks. Sufficient conditions are given to guarantee the impulsive consensus of the networked nonlinear dynamical system in directed networks with switching topologies. Furthermore how to design the impulsive control protocol is also presented.
- The future work is to consider the impulsive synchronization problem of networked nonlinear dynamical systems with stochastic topologies.

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Thank you!

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