静电场与高斯定理

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主要内容

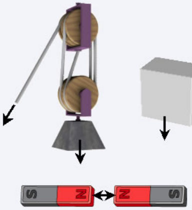
背景

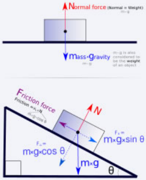
力学定律

$$\vec{F} = m\vec{a} \tag{1}$$

常见的力

推力　拉力　摩擦力　重力（万有引力）　...

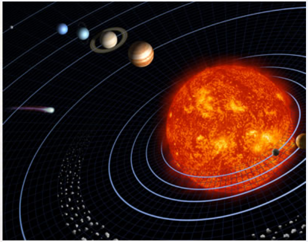




背景

基本相互作用力

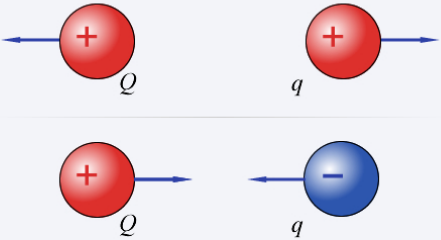
- 万有引力
- 电磁力
- 强作用
- 弱作用



静电力

库仑定律

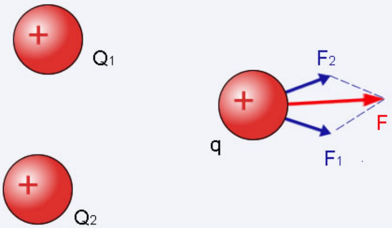
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\vec{n}_r \quad (2)$$



静电力

叠加原理

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots \tag{3}$$



超距作用or场

与常见的力不同：没有接触

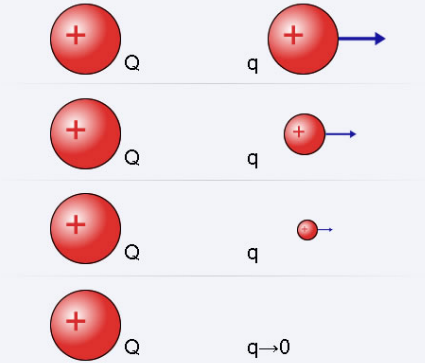
超距作用：电磁力不需要接触，一个电荷直接使其它电荷受力。

场：电荷周围伴随电场，电场使其它电荷受力。

电场强度

电场强度

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{n}_r q, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{n}_r = \frac{\vec{F}}{q}. \tag{4}$$

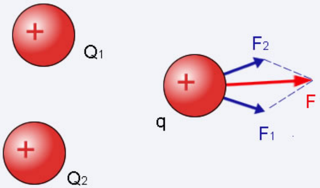


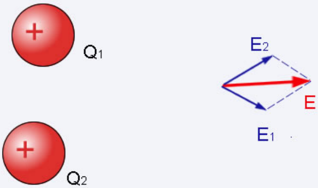
电场强度

电场强度的叠加原理

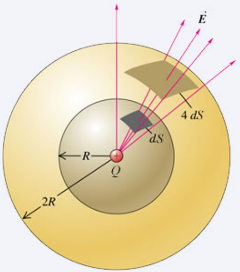
$$\vec{E} = \vec{F}/q = \vec{E}_1 + \vec{E}_2 + \ldots., \quad (5)$$

$$\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{n}_r. \quad (6)$$





电场通量

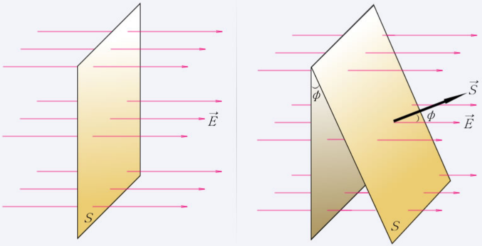


点电荷在球面上的电场通量

$$ES = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}4\pi r^2 = \frac{Q}{\varepsilon_0}, \tag{7}$$

$$EdS = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dS = \frac{Q}{\varepsilon_0}\frac{d\Omega}{4\pi}. \tag{8}$$

电场通量

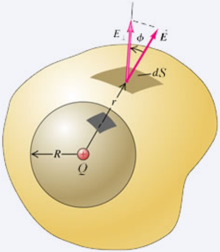


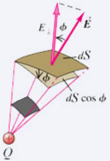
面元与场强不垂直

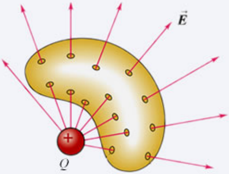
$$d\Phi = EdS\cos\phi, \tag{9}$$

$$d\Phi = \vec{E}\cdot d\vec{S}. \tag{10}$$

任意曲面的电通量





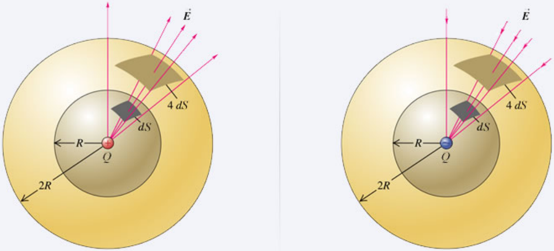


任意曲面的电通量

电荷在曲面内：$\Phi = \int E\cos\phi\, dS = \int \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$

电荷在曲面外：$\Phi = \int E\cos\phi\, dS = \int \vec{E}\cdot d\vec{S} = 0$

负电荷有相同的结果



任意曲面的电通量

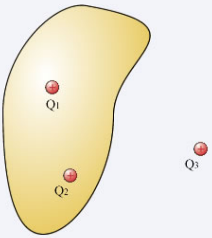
电荷在曲面内：$\Phi = \int E\cos\phi dS = \int \vec{E}\cdot d\vec{S} = \dfrac{Q}{\varepsilon_0}$

电荷在曲面外：$\Phi = \int E\cos\phi dS = \int \vec{E}\cdot d\vec{S} = 0$

高斯定理

高斯定理

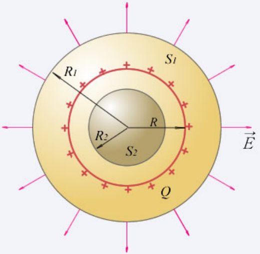
$$\begin{aligned}\Phi &= \int \vec{E}\cdot d\vec{S} = \int \vec{E}_1\cdot d\vec{S} + \int \vec{E}_2\cdot d\vec{S} + \ldots \\ &= \frac{Q_{in}}{\varepsilon_0}\end{aligned} \tag{11}$$



均匀带电球壳

例1：均匀带电球壳

电量Q均匀分布在半径为R的球壳上，分析其附近的电场强度。



$$\int_{S_1} \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} = E_1 S_1, \quad \Rightarrow \quad E_1 = \frac{Q}{4\pi\varepsilon_0 R_1^2}, \tag{12}$$

$$\int_{S_2} \vec{E}\cdot d\vec{S} = 0 = E_2 S_2, \quad \Rightarrow \quad E_2 = 0. \tag{13}$$

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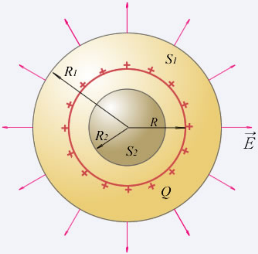

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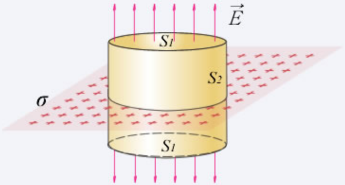
无穷大带电平板

例2：无穷大带电平板

无穷大平板上均匀分布着电荷，密度为σ.



$$\int_{S_1+S_1+S_2} \vec{E}\cdot d\vec{S} = \frac{S_1\sigma}{\varepsilon_0} = ES_1 + ES_1 + 0, \quad \Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}. \qquad (14)$$

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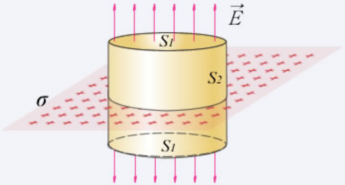

$$\int_{S_1+S_1+S_2} \vec{E}\cdot d\vec{S} = \frac{S_1\sigma}{\varepsilon_0} = ES_1 + ES_1 + 0, \quad \Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}. \tag{14}$$

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谢谢大家！！