What can We Learn from Analysis of the Financial Time Series?

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Here we report the research work about analysis of the financial time series based on nonlinear dynamics and statistical physics undertook in recent years by USTC complex system research group.

1. INVESTIGATION OF THE DISTRIBUTION AND SCALING OF FLUCTUATIONS FOR STOCK INDEX IN FINANCIAL MARKET

In order to probe the extent of universality in the dynamics of complex behavior in financial markets and to provide a basic and appropriate framework for developing economic models of financial markets, we investigated the distribution of the fluctuations in the Hang Seng index — the most important financial index in the Hong Kong stock market [1]. The data include minute by minute records of the Hang Seng index from January 3, 1994 to May 28, 1997. It was observed that the distribution of returns in the Hang Seng index shows apparent scaling behavior, which cannot be modeled by a normal distribution. The non-Gaussian dynamics of the stochastic process underlying the time series of returns of the Hang Seng index, is better modeled by a truncated Lévy distribution which is shown in Fig. 1. A power-law behavior is observed for the probability of zero return for time intervals Δt spanning at least two orders of magnitude. However, the power-law fall-off behavior in the tails deviate from that of Lévy stable process. The two tails of the distribution drop more slowly than a Gaussian, but faster than a Lévy process with an exponent outside the Lévy stable region. Especially after removing daily trading pattern from the data, the exponential deviation behavior from Lévy stable process is more clearly. The daily pattern thus affects strongly the analysis of the asymptotic behavior and scaling of fluctuation distributions. The exponential truncation ensures the existence of a finite second moment. The observations are useful for establishing dynamical models of the Hong Kong stock market [1].

2. BUILD A FINANCIAL MARKET MODEL BASED ON SELF-ORGANIZED PERCOLATION

The economy has been perceived as a collection of nonlinear interacting units. This collection is complex; everything depends on everything else. Physicists are looking for empirical laws that can reveal such complex interactions and theories that will help understand them [2-5]. As far as the financial markets are considered, due to intensive statistical studies during the last decade, the model of market fluctuation proposed by Bachelier in 1900 suffers the impugnation and the challenge of actual financial data such as the real-life markets are of return distributions displaying peak-center and fat-tail properties [6-7], one can observe volatility clustering and a non-trivial "multifractal" scaling [8-10], and so on. These universal features portray a world of non Gaussian random walks and inspire scientists to construct microstructure market models, such as Cont-Bouchaud model [11], Lux-Marchesi model [12], LeBaron model [13] and so on, to explain its underlying mechanisms. Furthermore, this key problem about what is underlying market mechanisms, is still open.

We focus on it for years and reap large profits from establishing and analyzing our market models including a activating

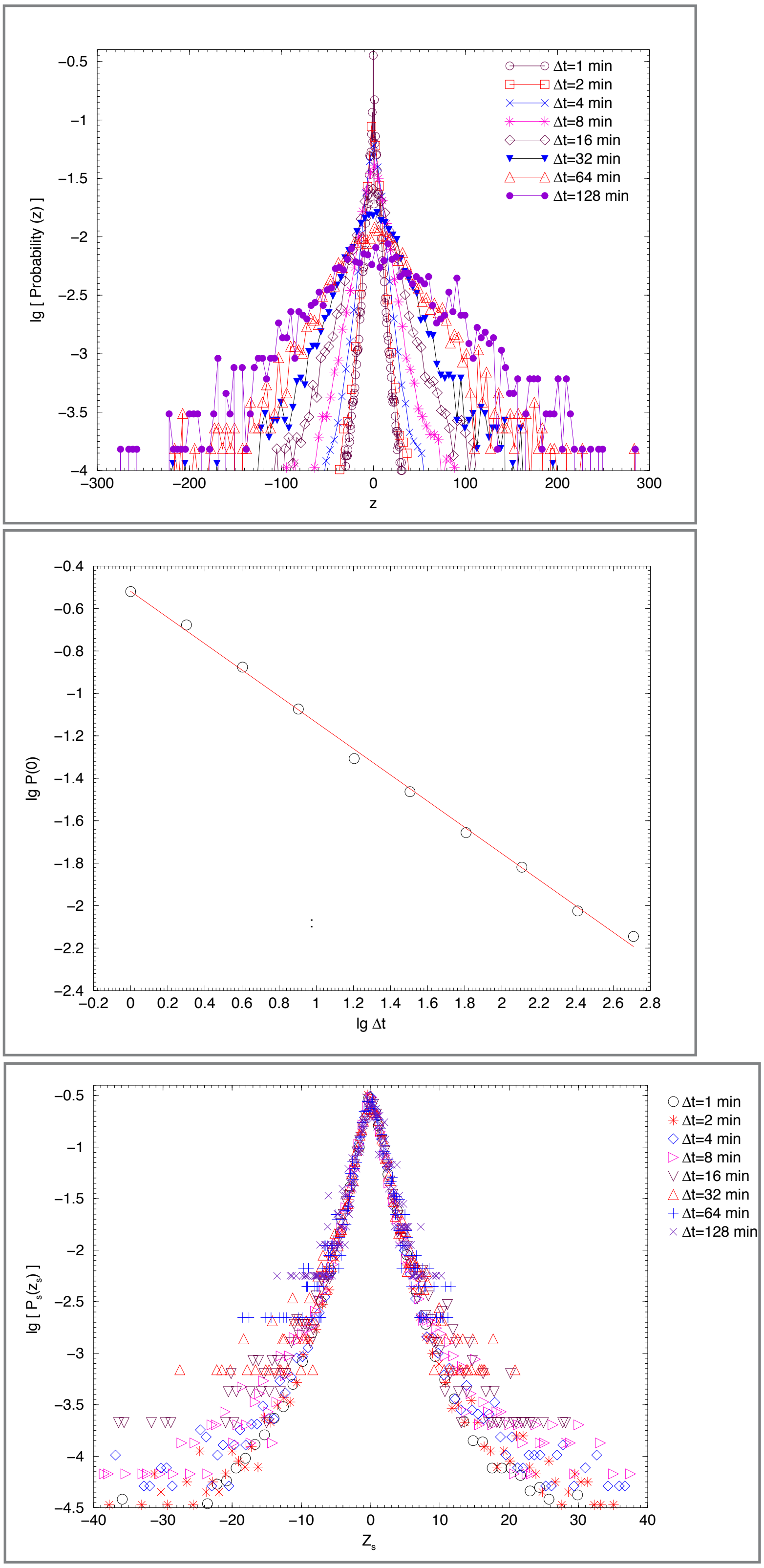

Fig. 1: Probability distributions of the returns and their scaling behavior of the Hang Seng index in Hong Kong stock market for the period January 3, 1994 to May 28, 1997. (a) The probability distributions of index returns for time separation Δt = 1, 2, 4, 8, 16, 32, 64, 128 min. (b) The central peak value P (0) as a function of Δt. A power-law behavior is observed. The slope of the best-fit straight line is -0.618 ± 0.025 from which we obtain the scaling exponent $\alpha = 1.619 \pm 0.05$ characterizing the Lévy distribution. (c) Re-scaled plot of the probability distributions shown in (a). Data collapse is evident after using rescaled variables with $\alpha = 1.619$. The abscissa is for the re-scaled returns, the ordinate is the logarithm of re-scaled probability.

model of individual behavior towards economics complex system and a stock market based on "Genetic Cellular Automata" with information exchange among individuals [14-15]. Based on them, considering the self-organized dynamical evolution of the behavior of investors and their structure, we build an agent based model to describe financial markets. It has incorporated the following components: (1) the behavior of investors evolve constantly according to excess demand; (2) As reality, the circle of professionals and colleagues to whom a trader is typically connected evolves as a function of time: in some cases, traders follow strong herding behavior and their effective connectivity parameter p is high; in other cases, investors are more individualistic and smaller p seems more reasonable. So investors structure (the complex interactions between traders) undergoes generational metabolism process repeatedly; (3) The effect of "herd behavior" on the trade-volume and the impact of each invest-cluster's trade-volume on the price are nonlinear. While this artificial stock market evolving, the number of investors participating in trading isn't constant; the network made up of invest-clusters takes on different structure; cooperation and confliction among invest-clusters are always operating; the affection of the herd behavior on the trade-volume varies dynamically accompanying the evolutionary of investor structure. In a word, the financial market is perceived as a complex system in which the large-scale dynamical properties depend on the evolutionary of a large number of nonlinear-coupled subsystems.

This model can iterate for a period of any length. More simulations have been done indicating that the return distribution of the present model obeys Lévy form in the center and displays fat-tail property, in accord with the stylized facts observed in real-life financial time series. Furthermore, this model reveals the power-law relationship between the peak value of the probability distribution and the time scales in agreement with the empirical studies on the Hang Seng Index [16]. It also achieves same avalanche dynamics and multi-fractal scaling properties of price changes as the real [17-18]. All the results indicate that underlying market mechanisms maybe is the self-organized dynamical evolution of the behavior of investors and their structure.

3. MODELING STOCK MARKET BASED ON GENETIC CELLULAR AUTOMATA

In the paper [14], an artificial stock market based on genetic cellular automata is established. Cells are used to represent stockholders, who has the capability of self-teaching and are affected by the investing history of the neighboring ones. The topological structure of CA in this paper is a two-dimensional square lattice with periodic boundary conditions. Before a trade, each stockholder should choose the trading strategies: to buy, to sell or to ride the fence. The stockholder's decision includes two steps: first, each stockholder works out a preparatory decision according to the history of its investment and the stock price. The stockholders of different risk-properties have different decision methods. The risk-neutral individuals directly inherit the last decision. The risk-aversed individuals' investing strategy is to buy at a low price and to sell at a high price. The individuals' risk-properties are given randomly initially, and can change along with the evolvement of the stock market. Similar to genetic algorithm [19], the risk property and the decision of each individual are naturally divided into four types of genes logically, including risk-property, deal-decision, price-decision and amount-decision. Each individual prefer to choose one of its successful neighbors to do crossover operation. The buyer with higher price and the seller with lower price will trade preferentially, and the trading-price is the average of seller's and buyer's price. The stock price is the weighted average of trading-price according to the trading-amount.

Simulation results about time series of price and returns based on genetic cellular automata show that when the proper initial condition and parameters have been chosen, the artificial stock market can generate its stock price whose trend and fluctuations are rather similar to that of real stock market [14]. In addition, in accordance with the empirical study on S&P500 [7] and Hang Seng index [1], the central part of the probability distribution of price returns in this model can be well fitted by a Levy distribution, while its tail is really fat as shown in Fig. 1 [1, 14].

4. EVOLUTIONARY PERCOLATION MODEL OF STOCK MARKET WITH VARIABLE AGENT NUMBER

The financial market has been proved to be a very important platform for the research of the "Complex System" field. By detailed analysis of the financial market price, more and more universal properties which are similar to those observed in physical systems with a large number of interacting units are discovered. The motivation to capture the complex behavior of stock market prices and market agents leads lots of sophisticated models based on different theoretical principles and evolutionary mechanics such as behavior-mind model, dynamic-games model, multi-agent model and so on. All of these models could reproduce some of the stylized observations of real markets, but fail to account for either the origin of the universal characteristics or some very important properties of the real multi-agent system.

To solve these problems, our new model is based on the following principles which were ignored by the former research [20]: (1) The system is an open system which allows the agents get into or get out of it; (2) The growth of the clusters is a gradually accumulating process controlled automatically by the model itself. (3) The "herd behavior" is magnified by self-organized accumulating rather than by adjusting the parameters forcibly; (4) By cooperation or conflict, the clusters could get even bigger or crushed.

To initialize our model, a lattice is taken up randomly. The interconnected nodes form a cluster. Every cluster could have three strategies: buying, selling and sleeping. During every iteration, some new agents would get into this system first. If the former strategies are success, the clusters would stay the same strategies with a higher probability. Otherwise, they would change their strategies with a higher probability. Not only the former strategies would affect the clusters' status, but also the neighbor clusters' status would have an effect. The bigger the cluster is, the higher influence on neighbor it owns. If the neighbor clusters take the same strategies, they would form a bigger cluster. On the other hand, the bigger cluster would have a higher probability to collapse. Because in the real world, the market would become much more danger when everyone takes the same strategy. Finally, the price would be determined by the overall status of the clusters. If the number of the clusters which take "buying" is bigger than "selling", the price increases, vice versa. In the next iteration, the fluctuation of the price would have a feedback of the clusters new status.

The simulation results which agree well with the reality are convincing support to our original ideas to some extent [20]. From the analysis based on this model, we learn that the causes of the various statistical properties of the real market are: the dynamical evolvement of the trader groups, i.e., the process in which different trader groups cooperate and conflict; the gradually accumulating process of the clusters' growth controlled by the model automatically; the self-organized accumulating effect on the magnified process of the "herd behavior".

5. EMPIRICAL STUDY ON THE VOLATILITY OF THE HANG-SENG INDEX

The volatility quantifies the activity of stock markets, defined as the number of transaction per unit of time connecting with interest of trades and is also the key input of virtually all option pricing models, including the classic Black and Scholes model [21] and the Cox-Ross-Rubinstein binomial model [22] that are based on the estimates of the asset volatility over the remaining life of the option. Without an efficient volatility estimate, it would be difficult for trades to identify situation in which options appear to be underpriced or overpriced. We study the statistical properties of volatility of minite-by-minite price fluctuation of Hang-Seng index in Hong Kong stock market [23].

The volatility is measured by locally averaging over a time window, the absolute value of price change over a short time interval. Define the price change as the difference between two successive logarithms of the index. In our work [23], we have found that the cumulative distribution of volatility is consistent with the asymptotic power-law behavior, characterized by power exponent μ=2.12, different from previous studies as μ=3. The volatility distribution remains the same asymptotic power-law behavior for the time scale from Δt =10 min to Δt = 80 min. We can find that Hong Kong stock market is more uncontrolled for the investors compared with other stock markets, thus the concussion (or impact) introduced into this market by the investors is stronger. Furthermore, we investigated the volatility correlations via the power spectrum and DFA (detrended fluctuation analysis) after filtering the effect caused by the daily oscillation pattern. Both of these two methods convincingly demonstrate the existence of long-range correlations. These scaling properties of the volatility distribution suggest that the volatility correlation is a possible explanation of the scaling behavior for price change distribution observed in [1].

6. ENLIGHTENMENT FROM VARIOUS CONDITIONAL PROBABILITIES ABOUT HANG SENG INDEX IN HONG KONG STOCK MARKET

In the analysis of the daily Hang Seng index of the Hong Kong stock market, two kinds of sign sequences as given conditions have been used to predict the future price movements. One is the parameter of multi-fractal spectrum Δf based on the indexes recorded in every minute, and the other is the variation of the close index Δf. Results show that correlation between large fluctuations of the close price and the condition in these two methods is strong and some sign sequences of the parameter Δf can be used to predict the probability of the near future price movements.

The efficient market hypothesis (EMH) indicates that if the market price were predictable, then the opportunities would be exploited to make a profit so that such opportunities would disappear in a competitive and efficient market. The proponents of the EMH thought that the market prices should behave like a random walk and the past price alone could not be used to predict the future price movement. However, many empirical observations cannot be explained by the EMH. The variation of the price is correlated and is not totally unpredictable.

In our work [24] , the Hang Seng stock index time series (from January 3, 1994 to May 28, 1997, totally 838 trading days, record by each minute) has been analyzed using various conditional probabilities to predict the index variation. It is found that the change of the close indexes is statistically correlated to the simple sign sequences of the close index variations in previous several days and to the sign sequences of the daily multi-fractal spectrum parameter Δf in previous several days, and the correlation is strong in the latter.

7. POWER LAW DISTRIBUTION OF WEALTH IN POPULATION BASED ON A MODIFIED EQUILUZ-ZIMMERMANN MODEL

Empirical evidence of the power-law distribution of wealth has recently attracted a lot of interest of economists and physicists. Taking into account the crowding and information transmission in financial market, Equíluz and Zimmermann (EZ) proposed a toy model ro reproduce the power-law wealth distribution [25]. The EZ model gives a power law distribution with an exponential cutoff that vanishes only for a particular parameter [26-28]. However, the real market is not able to subtly tune this parameter to a specific value.

In the work [29], we proposed a money-based model containing N units of money, where N is conserved. Then the total wealth is allocated to M economic entities, where M is variable. For simplicity, we may choose the initial state containing just N corporations, each with one unit of money. The state of this system is mainly described by $n(s)$, which denotes the number of cooperation owning s units of money. At each time step, we randomly select a unit of money from the wealth pool. Since it must belong to a certain corporation, we in this way select an economic entity too. The evolution of the system is under the following rules. (1) With probability $1-a$, another unit of money is randomly selected. If the two selected units are occupied by different corporations, then the two corporations with all their money combine into one entity; otherwise, no combination. (2) With probability $a\gamma/s$, the economic entity that owns the selected money is dissociated; here s is the amount of capital owned by this corporation, and $a\gamma$ reflects the dissocialized (bankruptcy) possibility of any economic entity. After disassociation, these s units of money are simply assumed to be redistributed to s new companies, each with just one unit. (3) With probability a $(1-\gamma/s)$, nothing is changed.

The major difference between our model and the EZ model is that the dissocialized probability of an economic entity, after being chosen, is proportional to $1/s$ in our model (to take into account the fact that a larger corporation can live longer than a smaller one statistically), while in the EZ model, the corresponding probability is simply proportional to 1. Still, it must be stressed that in the EZ model a power-law distribution without exponential correction is obtained only for a particular parameter, while our model will give the exact power-law distribution in a wide range of a. The power-law exponent depends on the model parameter in a nontrivial way and can be exactly calculated [29].

8. SELF-SEGREGATION AND ENHANCED COOPERATION IN AN EVOLVING POPULATION THROUGH LOCAL INFORMATION TRANSMISSION

In our work [30] we introduce the local information in the evolutionary minority game (EMG) and studied the effects of local information transmission and imitation among agents in an evolving population.

Showing the sharp comparison with EMG, evolution in our model is made through local information transmission among agents. In order to incorporate the idea of neighboring agents, we arrange all the agents on a ring. Each agent has two nearest neighbors and knows the wealth and the strategy parameter p-values of her two neighbors. The better neighbor at the time step t is the one with the higher accumulative wealth. At each time step, each agent compares her wealth with her nearest-neighbors. If the agent has less accumulative wealth than her better neighbor, she modifies her p-value by choosing a new p-value randomly within a range R centered on the p-value of her better neighbor. As shown in Fig. 2(a), the steady states distribution $P(p)$ for different R are symmetric about $p = 0.5$ with peaks around p=0 and p=1, a behavior also exhibited in EMG. The dependence of the standard deviation (SD) on R is not monotonic. It shows a minimum at $R = R_c \approx 0.0065$ which is significantly lower than the random coin-toss limit. The results indicate that at R_c, there is an optimal degree of correlation between the modified p-value and her neighbor's p-value upon evolution through local information exchange and imitation. Fig. 2(b) shows the SD for the present model as a function of m for a system with N=101 and R = 0.0065. The results indicate that the SD is insensitive to m and takes on a value significantly lower than that of the EMG and other models. The average success rate in the present model for m=4 and R=0.0065 is about 0.4932, while the EMG saturates at a success rate of 0.4713.

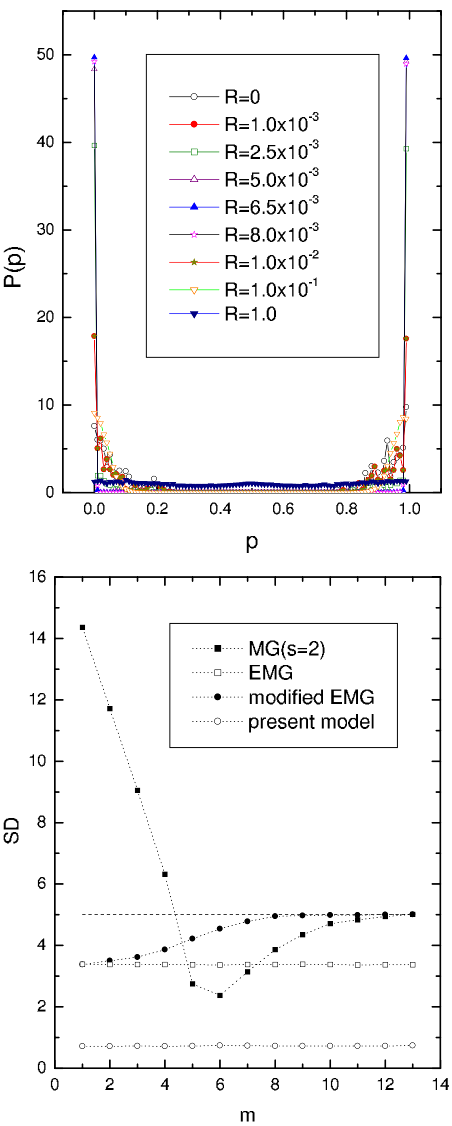

Fig. 2: (a) The distribution P(p) for different values of R (m = 4; N = 101). Data are obtained by averaging over 100 independent runs of 1×10^7 time steps. (b) The standard deviation (SD) as a function of m for the present model (R = 0:0065), EMG (d = −4; R =0:2), and MG (s = 2), in a system with N = 101. Data are obtained by averaging over 100 independent runs of 1×10^7 time steps. The horizontal dashed line gives the random coin-toss limit corresponding to a population with randomly deciding agents.

In summary, we have proposed and studied numerically a new version of the evolutionary minority game in which local information transmission among neighboring agents is allowed. It is found that imitation reduces the standard deviation greatly and thus leads to an enhanced average success rate for the competing population as a whole.

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