2009-5-3

Reading Cleaning up Mystery-the Original Goal:

Diffusion:

$$\begin{cases} \frac{\partial u}{\partial t} + div(J) = 0 \\ J = -D \cdot grad(n) \end{cases} \qquad (1)$$

Rayleigh-Jeans equipartition law:

$\begin{pmatrix} Energy\ of\ radiation\ per\ unit\ volumn \\ with\ the\ frequencies\ between\ v\ and\ v + dv \end{pmatrix} = \frac{8\pi}{c^3} v^2 kTdv$ can give the mean square value of the instantaneous VELOCITY that a particle is wilding fluctuating.

BUT this VELOCITY is not the VELOCITY that we are seeking.

Divergence $div\vec{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

Where $\vec{A} = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$

Here is only one dimention, so we can simply note

$div(J) = \frac{\partial J}{\partial x}$

and $J = -D \cdot grad(n)$

where $grad(\cdot)$ is a notation of gradient operator, also can be written as $\nabla(\cdot)$

$$grad\,\vec{n}(x, y, z) = (\frac{\partial \vec{n}}{\partial x}, \frac{\partial \vec{n}}{\partial y}, \frac{\partial \vec{n}}{\partial z})$$

here we only consider one dimension,

so $grad(n) = \frac{\partial n}{\partial x}$

where n is the concentration.

Also, we have the relationship

$$J = n \cdot v \qquad (2)$$

$\langle z \rangle = x \qquad \langle y \rangle = x$

$$\begin{matrix} z & x & y \\ x(t-\tau) & x(t) & x(t+\tau) \\ before & now & future \end{matrix}$$

$$\bar{v} = \frac{x(t+\tau) - x(t-\tau)}{2\tau} \qquad (3)$$

$$P(y \mid x, I) = A\exp\left[-(y-x)^2 / 2\sigma^2(\tau)\right] \tag{4}$$

From Lecture Notes in Statistics, $P(H, D \mid I) = P(D \mid I)P(H \mid D, I) = P(H \mid I)P(D \mid H, I)$

We can get here $P(z, x \mid I) = P(z \mid I)P(x \mid z, I) = P(x \mid I)P(z \mid x, I)$ (5)

And $P(x \mid I) = const$,

$P(z \mid x, t, I) = A \cdot P(z \mid I) \cdot P(x \mid z, I)$, where $A = \dfrac{1}{P(x \mid I)}$ (6)

The prior probability $P(z \mid I)$ is clearly proportional to $n(z)$,

And from (4), $P(x \mid z, I) = A\exp\left[-(z-x)^2 / 2\sigma^2(\tau)\right]$

We can get

$$\log P(z \mid x, I) = \log n(z) - (z-x)^2 / 2\sigma^2(\tau) + (const) \tag{7}$$

The maximum probability of $P(z \mid x, I) = 1$, so $\log P(z \mid x, I) = 0$

Then from (7), we can get $\log n(z) - (z-x)^2 / 2\sigma^2(\tau) + (const) = 0$

Do partial differential about x,

$$\frac{\partial}{\partial x}(\log n) - \frac{\partial}{\partial x}\left((z-x)^2 / 2\sigma^2(\tau)\right) + \frac{\partial}{\partial x}(const) = 0$$

$$\frac{\partial}{\partial x}(\log n) - 2(z-x) \cdot \left(-\frac{\partial x}{\partial x}\right) \Big/ 2\sigma^2(\tau) = 0$$

In fact, $\dfrac{\partial}{\partial x}$ is one-dimensional gradient operator.

$$(z-x) \cdot \big/ 2\sigma^2(\tau) = grad(\log n)$$

The most probable value of the past position z is not x, but

$$\hat{z} = x + \sigma^2 grad(\log n) = x + (\delta x)^2 grad(\log n) \tag{8}$$

Whereupon, substituting into (3) we estimate the drift velocity to be

$$\bar{v} = \frac{\langle x(t+\tau)\rangle - \langle x(t-\tau)\rangle}{2\tau} = \frac{x - \hat{z}}{2\tau} = \frac{x - x - \sigma^2 grad(\log n)}{2\tau} = -(\delta x)^2 / 2\tau\, grad(\log n) \tag{9}$$

and our predicted average diffusion flux over the time interval 2τ is

$$J(x,t) = n \cdot \bar{v} = -(\delta x)^2 / 2\tau\, grad(n) \tag{10}$$

Bayes' theorem has given us just Einstein's formula for the diffusion coefficient:

$$D = \frac{(\delta x)^2}{2\tau} \tag{11}$$