Integral bounded by supremum of second derivative

Let f'' be continuous on $[a, b]$, and $f(a) = 0 = f(b)$, then integration by parts formula gives

$$\int_a^b f(x)\,dx = \frac{1}{2}\int_a^b f''(x)\,(x-a)\,(x-b)\,dx;$$

thus

$$\left|\int_a^b f(x)\,dx\right| \le \frac{(b-a)^3}{12}\max_{a\le x\le b}\left|f''(x)\right|.$$