A Note for P308 Smoller---Starlike curve

The curve

$$u - u_1 = -\sqrt{(v - v_1)(p(v_1) - p(v))} = s_1(v; U_1), \; v < v_1.$$

is starlike w.r.t. the point U_1, i.e. any ray through U_1 meets this curve at most one point.

Indeed, any ray through U_1 has the form

$$l_k : u - u_1 - k(v - v_1).$$

The intersection of l_k with $s_1(v; U_1)$ satisfies

$$\begin{cases} u - u_1 = -\sqrt{(v - v_1)(p(v_1) - p(v))}, \\ u - u_1 = k(v - v_1). \end{cases} \qquad (*)$$

Thus

$$k^2 (v - v_1)^2 = (v - v_1)(p(v_1) - p(v)).$$

And

$$k^2 = -\frac{p(v) - p(v_1)}{v - v_1}.$$

If two points (v_1, u_1), (v_2, u_2) $(v_2 < v_1 < v_1)$ satisfies (*), then

$$\frac{p(v_1) - p(v_1)}{v_1 - v_1} = \frac{p(v_2) - p(v_1)}{v_2 - v_1} = \frac{p(v_2) - p(v_1)}{v_2 - v_1}.$$

Lagrange intermediate theorem tells us

$$p'(v_{11}) = \frac{p(v_1) - p(v_1)}{v_1 - v_1} = \frac{p(v_2) - p(v_1)}{v_2 - v_1} = p'(v_{12})$$

for some $v_{11} \in (v_1, v_1)$, $v_{12} \in (v_2, v_1)$.

Hence Rolle's theorem implies that

$$p''(v_{12}) = 0$$

for some $v_{12} \in (v_2, v_1)$.

A contradiction to the fact $p''(v) > 0$. The proof is thus complete.

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