Simplification of a Model

for conservation laws and consequences

We consider the following scalar conservation laws:

$$\begin{cases} u_t + f(u)_x = 0, & t > 0, x \in R; \\ u|_{t=0} = u_0, & x \in R. \end{cases} \qquad (*)$$

where $u_0 \in L^\infty(R)$ and $f'' > 0$ on $co(Range(u_0))$.

In further investigation, we may assume

$$f(0) = 0 = f'(0).$$

Indeed, let c be the unique critical point of f, i.e.

$$f'(c) = 0.$$

Then solving (*)1 is equivalent to solving

$$v_t + F(v)_x = 0$$

with $F(v) = f(v + c) - f(c)$.

- $v = u - c$;
- Existence and uniqueness of entropy solutions;
- $F(0) = f(c) - f(c) = 0$;
- $F'(0) = f'(c) = 0$;
- $F''(v) = f''(v + c) > 0$.

Remark 1 In the book

Shock waves and Reaction-Diffusion Equations

written by Joel Smoller, it is proved that (see P295)

Theorem Assume more that $f(0)=0=f'(0)$, $spt\ u_0$ is compact, then the entropy solution u of (*) decays to 0 as $t\to\infty$, uniformly in x, at a rate $t^{-\frac{1}{2}}$.

Thus the above analysis gives:

Theorem Assume more that $spt\ u_0$ is compact, then the entropy solution u converges to the unique critical point of f, as $t\to\infty$, uniformly in x, at a rate $t^{-\frac{1}{2}}$.

Remark 2 We know Morse theory is concerned with the critical point of a map, around where the topology. How does this scalar or more generally, systems of conservation of laws related to Morse theory? This would be an interesting interacting field.

Remark 3 It is a little stupid. Since it took me two days to solve. I've asked some mates, but I was not satisfied with their answers.

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