Rest point and asymptotic limit

Theorem Let $x(t)$ be a solution of

$$x'(t) = \phi(x(t)),$$

and

$$\lim_{t\to\infty} x(t) = x_0 \text{ (asymptotic limit)},$$

then

$$\phi(x_0) = 0 \text{ (rest point)}.$$

Proof If $\phi(x_0) \neq 0$, then the flow $x(t)$ take a small neighborhood U of x_0 into V such that $V \cap U = \varnothing$.

This means precisely that

$$\exists U, \text{ s.t. } \forall\ t, \exists\ s > t, x(s) \notin U.$$

A contradiction to

$$\lim_{t\to\infty} x(t) = x_0.$$