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Brief paper

A remark on collective circular motion of heterogeneous multi-agents☆

Zhiyong Chen [a,1], Hai-Tao Zhang [b,2]

[a] School of Electrical Engineering and Computer Science, The University of Newcastle, Callaghan, NSW 2308, Australia

[b] Key Laboratory of Image Processing and Intelligent Control, Department of Control Science and Engineering, State Key Laboratory of Digital Manufacturing Equipments and Technology, Huazhong University of Science and Technology, Wuhan, 430074, PR China

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ABSTRACT

The study on collective circular motion has attracted researchers over the past years. Many control algorithms have been successfully developed for achieving various circular motion patterns. However, the existing algorithms rely on more or less global information including a reference beacon, a common reference frame, agent labels, or agent homogeneity. In this paper, an improved algorithm is proposed for a group of heterogeneous agents, not relying on any of the aforementioned global information. The algorithm is supported by analytical analysis and verified in both simulation and experiments.

1. Introduction

Collective behaviors are often encountered in natural, social and engineering systems. The particularly interesting collective behavior studied in this paper is that, individuals perpetually rotate around a real or virtual center, which is called *collective circular motion* or *torus*. It can find many examples in various natural systems like foraging ants around a piece of rice, a swirlingly growing epiphyte colony, and panic escaping fish school around a predator (Couzin, Krause, James, Ruxton, & Franks, 2002; Vicsek, 2008). From the engineering point of view, due to the high cost and low flexibility to maintain a centralized controller, a collective controller should be decentralized and independent of global information. For instance, we expect to propose a control algorithm which does not rely on a common reference frame, a reference beacon, a group leader, agent labels, or agent homogeneity. However, tracing back into the literature, we find it is actually a challenging task to construct such an algorithm free of any global information. For example, one of the earliest contributions was given in Leonard and Friorelli (2001), where circular motions are obtained with a virtual reference beacon. Following this line, more control algorithms were developed to gain collective stable circular motions with allowable equilibrium configurations (Justh & Krishnaprasad, 2004; Paley, Leonard, & Sepulchre, 2004). Circular motions were also studied in the scenario of cyclic pursuit in Jeanne, Leonard, and Paley (2005), Marshall, Broucke, and Francis (2004), Pavone and Frazzoli (2007) and Chen and Iwasaki (2008) etc., where no virtual reference beacon is required. In particular, a group of mobile agents was studied in Pavone and Frazzoli (2007) where each agent pursues the leading neighbor along the line of sight rotated by a common offset angle, resulting in a circular motion. The cyclic pursuit formation is based on a fixed network topology, especially, represented by a circulant matrix. The result was extended in Ren (2009) by introducing a more general rotation matrix for double integrator dynamics. Along this research line, the latest work is referred to in Lin and Jia (2010) and Lin, Qin, Li, and Ren (2011) where control protocols were proposed to make all agents surround a common point with a desired formation structure, in 2D and 3D spaces, respectively. A technical feature in the Refs. (Li, Liu, Ren, & Xie, 2013; Lin & Jia, 2010; Lin et al., 2011;

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E-mail addresses: zhiyong.chen@newcastle.edu.au (Z. Chen), zht@mail.hust.edu.cn (H.-T. Zhang).

[1] Tel.: +61 2 4921 6352,; fax: +61 2 4921 6993.

[2] Tel.: +86 27 8755 9720; fax: +86 27 8755 9416.

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Pavone & Frazzoli, 2007; Ren, 2009) is the analysis of the Laplacian corresponding to a fixed network topology, which requires the individuals to be labeled. Recently, there was some work on time varying communication graphs. For instance, a framework on stabilization of planar collective motion was provided in Sepulchre, Paley, and Leonard (2008) which includes circular formations. But the work was based on identical steered particles moving in a plane with a uniform constant speed. Another control method was studied in Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008) which guarantees the global asymptotical stability of the circular motion. In such a configuration, however, a leader exists who always knows the position of the reference beacon. More recently, we proposed a leader-free no-beacon algorithm for generating a collective circular motion behavior in Chen and Zhang (2011), and showed that, a stable torus phenomena emerges under a certain uniformly jointly connected condition. The condition was further studied in Chen and Zhang (2012). In this sense, the generality of collective circulation control has been substantially improved.

However, the aforementioned torus control algorithms, including our previous work (Chen & Zhang, 2011), are still not completely decentralized, relying on more or less global information. It motivates the research of this paper, which follows the research line set in Chen and Zhang (2011). In particular, it is required in Chen and Zhang (2011) that, each agent has a same rotational radius and an identical reference frame (e.g., the earth). Therefore, we aim to propose an improved controller to further remove these requirements. This improvement may generalize the potential applications of the collective circular motion controller. The main features of the improved controller include: (i) each agent does not require the motion radii of its neighbors; (ii) each agent makes its motion decision solely based on the relative distances and the relative moving directions with respect to its neighbors. As a result, this work is expected to help reveal the interactive mechanism behind more natural collective circular motion behaviors and to be more practical in engineering applications.

2. Problem formulation and preliminaries

Consider a group of $n \geq 2$ nonholonomic agents moving in a planar space, each of which has the velocity of $\vec{v}_i \in \mathbb{R}^2$. Let $\vec{v}_i := v_i[\cos\theta_i, \sin\theta_i]^\mathsf{T}$ and $v_i = \|\vec{v}_i\|$ where v_i and θ_i are the velocity magnitude and direction, respectively. Let $p_i := [x_i, y_i]^\mathsf{T}$ be the agent's Cartesian coordinate in a fixed reference frame Σ. Then, we have the following dynamics for each individual agent:

$$\dot{x}_i = v_i\cos\theta_i, \qquad \dot{y}_i = v_i\sin\theta_i, \qquad \dot{\theta}_i = \omega_i,$$
$$i \in \mathbb{N} := \{1, 2, \ldots, n\}, \tag{1}$$

where ω_i is the rotational speed. In Chen and Zhang (2011), the n agents are assumed homogeneous in the sense that they, when isolated, share the same linear and rotational speeds, i.e.,

$$v_i = v_o, \qquad \omega_i = \omega_o, \quad i \in \mathbb{N}$$

and hence the same circular motion radius $r = v_o/\omega_o$. The common radius is the global information for all agents. In this paper, we will further consider the case where the agents are heterogeneous and they do not share any global information. More specifically, we assume an isolated agent has linear and rotational speeds

$$v_i = v_{oi}, \qquad \omega_i = \omega_{oi}, \quad i \in \mathbb{N}$$

for some constant v_{oi} and ω_{oi}, and hence a circular motion radius $r_i = v_{oi}/\omega_{oi}$. We assume ω_{oi}'s have a finite resolution, therefore, there exists a constant ω_o such that $\omega_{oi} = \ell_i\omega_o$ for different integers ℓ_i's.

For notational convenience we define

$$p_{ij} := p_i - p_j, \qquad \text{i.e., } x_{ij} := x_i - x_j, \qquad y_{ij} := y_i - y_j$$

as the relative position between two agents. For each individual agent, we define its neighborhood with a radius ρ. More precisely, for a complete position distribution $p = \mathrm{col}(p_1, \ldots, p_n)$, the i-th agent has a neighborhood

$$\mathcal{N}_i(p) := \{j \in \mathbb{N} \mid \|p_{ij}\| < \rho\}.$$

This neighborhood reflects the distance limit of communication between two agents. Let $r = \max_{i\in\mathbb{N}} r_i$ and assume $\rho > 2r$. We cite the definition of collective circular motion from Chen and Zhang (2011) with a slight modification to accommodate heterogeneous agents as follows.

Definition 2.1. A trajectory $p(t)$ of the group of agents (1) is called a *collective circular motion* if there exist $T_i > 0$, $r_i > 0$, and $q_o = [\xi_o, \zeta_o]^\mathsf{T}$, such that,

$$p_i(t + T_i) = p_i(t), \qquad p_i(t) - q_o = r_i[\cos\phi_i(t), \sin\phi_i(t)]^\mathsf{T},$$
$$\dot{\phi}_i(t) \geq 0, \; i \in \mathbb{N}, \forall t \geq 0.$$

Clearly, in a collective circular motion, all agents move in circles around q_o with different radii r_i's. Without loss of generality, a counter-clockwise motion is considered here by assuming $\dot{\phi}_i(t) \geq 0$. The main feature of the controller proposed in this paper, compared with the existing one in Chen and Zhang (2011), is twofold. On one hand, the individual rotation information r_i of the agent i is unavailable for any other agent. On the other hand, the fixed reference frame Σ is unavailable for any agent. In fact, the agent i has its own reference frame Σ_i which is Σ rotated by an angle φ_i (see Fig. 1). It is noted that, Σ_i is not attached to the agent i but a constant reference frame seen by the agent i only. Therefore, the agent i can measure the following states:

(i) r_i and ω_i: its own rotational radius and speed;
(ii) $\vartheta_i := \theta_i - \varphi_i$: its own moving direction with respect to its reference frame Σ_i (the absolute angle θ_i is not assumed measurable; for instance, the agent may take the initial direction as 0, i.e., $\vartheta_i(0) = 0$ or $\varphi_i = \theta_i(0)$, then, $\vartheta_i(t)$ can be calculated as $\vartheta_i(t) = \int_0^t \omega_i(t)dt$);
(iii) $\theta_{ij} := \theta_i - \theta_j$: the relative moving direction between the agent i and its neighbor agent j;
(iv) $\wp_{ij} := R_i p_{ij}$: the relative position between the agent i and its neighbor agent j with respect to the reference frame Σ_i for $R_i := \begin{bmatrix} \cos\varphi_i & \sin\varphi_i \\ -\sin\varphi_i & \cos\varphi_i \end{bmatrix}$.

Remark 2.1. A simple calculation shows that

$$\begin{aligned}\wp_{ij} = R_i p_{ij} &= \|p_{ij}\| R_i[\cos\angle p_{ij}, \sin\angle p_{ij}]^\mathsf{T} \\ &= \|p_{ij}\|[\cos(\angle p_{ij} - \varphi_i), \sin(\angle p_{ij} - \varphi_i)]^\mathsf{T}\end{aligned}$$

where $\angle p_{ij}$ represents the angle of p_{ij} in the reference frame Σ. In other words, to measure $\wp_{ij}$ in (iv) is equivalent to measure the distance $\|p_{ij}\|$ and the angle of p_{ij} in the reference frame Σ_i, i.e., $\angle p_{ij} - \varphi_i$. □

In the existing work (Chen & Zhang, 2011), $r_i = r$ is a kind of global information to all agents, and the absolute values θ_i and θ_j are measurable by assuming $\Sigma_i = \Sigma$ is another kind of global information (or by assuming $\varphi_i = 0$). However, no global information is assumed in this paper. The main objective of this paper is to find a control algorithm for each individual agent in (1) using the feedback from its neighbors, say,

$$v_i = \sum_{j\in\mathcal{N}_i(p)} \kappa(r_i, \omega_i, \vartheta_i, \theta_{ij}, \wp_{ij}), \quad i \in \mathbb{N} \tag{2}$$

for some function κ, such that the trajectory $p(t)$ of the closed-loop system converges to a collective circular motion $\check{p}(t)$, i.e.,

$$\lim_{t\to\infty}(p(t) - \check{p}(t)) = 0. \tag{3}$$

It is worth emphasizing that, in the present scenario, the network communication topology always varies over time as every agent determines its neighbors based on the time varying relative distances. Also, the proposed algorithm focuses on speed control for v rather than steering control for ω to achieve a collective circular motion (the rotational speed controller $\omega_i = \omega_{oi}$ is simply used here). In fact, a proper steering controller for ω can be further incorporated for phase distribution (i.e., how the agents are distributed along the circles once a collective circular motion is formed) as studied in Chen and Zhang (2011) for homogeneous agents.

Next, we define a class of dynamics whose trajectories may converge to a collective circular motion. For convenience, we introduce a coordinate transformation, with $q = \mathrm{col}(q_1, \ldots, q_n)$,

$$\Phi : (p, \theta) \mapsto (q, \theta) = \begin{cases} \xi_i := x_i - r_i \sin\theta_i \\ \zeta_i = y_i + r_i \cos\theta_i, \end{cases}$$

$$q_i := [\xi_i, \zeta_i]^{\mathsf{T}}, \quad i \in \mathbb{N},$$

and a class $\mathcal{S}$ function

$$\mathcal{S}(x) = \left\{ f : [0, \infty) \mapsto [0, a] \mid f \text{ is Lipschitz continuous and } f(\tau) = \begin{cases} 0, & \tau \geq x \\ > 0, & \tau < x, \end{cases} 0 < a < \infty \right\}.$$

Throughout the paper, we call the dynamics

$$\dot{q} = -f(q, \theta) \tag{4}$$

collective circular motion dynamics if

$$f(q, \theta) := \mathrm{col}(f_1(q, \theta), \ldots, f_n(q, \theta)),$$
$$f_i(q, \theta) = \sum_{j \in \mathbb{N}, j \neq i} \alpha(\|p_{ij}\|) q_{ij}/4, \quad q_{ij} := q_i - q_j \tag{5}$$

under the coordinate transformation Φ and the function α belongs to the class $\mathcal{S}$, i.e., $\alpha \in \mathcal{S}(\rho)$.

Obviously, under the coordinate transformation Φ, the coordinate q_i represents the circular motion center of the agent p_i. So, the collective circular motion problem for p_i reduces to a consensus problem for q_i. The dynamics (4) are called collective circular motion dynamics because they govern trajectories which converge to a collective circular motion. This claim is supported by the following proposition. The special case with homogeneous agents ($r_i = r$) was studied in Chen and Zhang (2011), which can be easily extended to the following general case by using the same proof with a slight modification.

Proposition 2.1. *Consider the collective circular motion dynamics (4), i.e., $\dot{q} = -f(q, \theta)$. Suppose there exists an infinite time sequence $t_1 < t_2 < t_3 < \cdots$, such that, for any $[t_i, t_{i+1})$, $i = 1, 2, \ldots$, the trajectory $p(t)$ of (4) with the proximity net $G(p)$ (under $(p, \theta) = \Phi^{-1}(q, \theta)$) is jointly-connected with a uniformly bounded joint connectivity intensity. Then, there exists a collective circular motion $\check{p}(t)$, such that $p(t)$ asymptotically converges to $\check{p}(t)$, i.e., $\lim_{t\to\infty}(p(t) - \check{p}(t)) = 0$.* □

Now, an interesting design objective is to find the controller (2) such that the closed-loop system is governed by the collective circular motion dynamics (4). However, this objective is usually too strong to ask for. In fact, the closed-loop system of the form (4) implies

$$\dot{q}_i = -f_i(q, \theta) = -\sum_{j \in \mathbb{N}, j \neq i} \alpha(\|p_{ij}\|) q_{ij}/4,$$

and hence

$$\dot{p}_i = -\sum_{j \in \mathbb{N}, j \neq i} \alpha(\|p_{ij}\|) q_{ij}/4 + r_i \omega_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix} = v_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}.$$

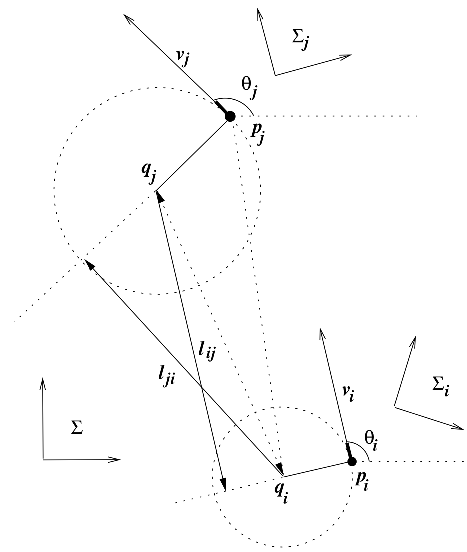

Fig. 1. The main control algorithm for an agent. The controller component μ_{ij} for the agent i is designed along the direction q_{ji} ($q_{ji} = q_j - q_i$, represented by the dotted arrow line from q_i to q_j) when q_j is available in Chen and Zhang (2011) and along the direction l_{ji} when q_j is unavailable in the present scenario. Similarly, the controller component μ_{ji} for the agent j is designed along the direction l_{ij} when q_i is unavailable in the present scenario.

But it is usually impossible to design v_i to make the vector $[\cos\theta_i, \sin\theta_i]^{\mathsf{T}}$ be parallel to the weighted summation of the vectors q_{ij} to satisfy the above equation. This impossibility is caused by the fact that the agent's dynamics are nonholonomic (v_i is a scalar). So we must turn to a more practical solution, that is, to find the controller (2) such that the closed-loop system is governed by approximate collective circular motion dynamics in a certain sense, which are rigorously described as follows. Let $\dot{q} = -f(q, \theta)$ be collective circular motion dynamics. Then, the dynamics $\dot{q} = -\mathcal{F}(q, \theta)$ are called *approximate collective circular motion dynamics* if the trajectory $q(t)$ satisfies

$$\bar{q}(t) - q(t) = O(1/\omega_o), \tag{6}$$

$$\dot{\bar{q}}(t) = -f(\bar{q}(t), \theta) + O(1/\omega_o) \tag{7}$$

for a signal $\bar{q}(t)$, where $O(1/\omega_o)$ represents the first order of smallness as $1/\omega_o \to 0$. Recall that ω_o is defined by $\omega_{oi} = \ell_i \omega_o$ for different integers $\ell_1, \ldots, \ell_n$.

In the above definition, if the smallness $O(1/\omega_o)$ is ignored, the dynamics $\dot{q} = -\mathcal{F}(q, \theta)$ governing $q(t)$ reduce to $\dot{q} = -f(q, \theta)$. The approximation is defined in this sense. As a result, we expect the trajectories of $\dot{q} = -\mathcal{F}(q, \theta)$ to be approximated by those of $\dot{q} = -f(q, \theta)$. In what follows, we will focus on the controller synthesis problem as defined below. Also, numerical simulation will run on the original system $\dot{q} = -\mathcal{F}(q, \theta)$ to illustrate the effectiveness of the approximation.

Controller synthesis problem: For the system (1), design the controller (2) such that the closed-loop system is governed by approximate collective circular motion dynamics.

3. Main results

As we have discussed in the previous section, the main objective of this paper is to design a decentralized speed controller such that the closed-loop system is governed by approximate collective circular motion dynamics. More specifically, we aim to design the forward motion speed v_i to achieve a special closed-loop system $\dot{q} = -\mathcal{F}(q, \theta)$ whose trajectories satisfy the conditions (6) and (7). The problem has been solved in Chen and Zhang (2011) for the special case with $r_i = r$ and $\Sigma_i = \Sigma$. Here we will deal with the general case.

Let $[\dot{x}_{di} \ \ \dot{y}_{di}]^\top$ be the desired velocity of the agent, then we have $v_i = [\cos\theta_i \ \ \sin\theta_i][\dot{x}_{di} \ \ \dot{y}_{di}]^\top$. Hence, we have the geometric constraint $[\dot{x}_{di} \ \ \dot{y}_{di}]^\top = [\cos\theta_i \ \ \sin\theta_i]^\top r_i\omega_i + \mu_i$ where μ_i is the desired velocity of the agent's circular center. This motivates a controller in the following form:

$$v_i = r_i\omega_i + [\cos\theta_i \ \ \sin\theta_i]\mu_i, \quad r_i = v_{oi}/\omega_{oi}.$$

The desired motion of the agent's circular center is $\mu_i = \sum_{j\in\mathbb{N}, j\neq i}\mu_{ij}$ with μ_{ij} being the influence caused the agent j in the neighborhood. When $r_i = r$, $i \in \mathbb{N}$, the control μ_{ij} was designed in Chen and Zhang (2011) as follows

$$\mu_{ij} = \alpha(\|p_{ij}\|)q_{ji} = -\alpha(\|p_{ij}\|)\begin{bmatrix} x_{ij} - r(\sin\theta_i - \sin\theta_j) \\ y_{ij} + r(\cos\theta_i - \cos\theta_j)\end{bmatrix}.$$

Geometrically speaking, the agent i first finds the circular motion center of the agent j and makes its own motion center move toward that of the agent j (i.e., along the direction q_{ji} represented by the dotted arrow line from q_i to q_j in Fig. 1). However, this scheme is not valid when r is not a common information. In particular, the agent i does not know the motion radius r_j or the center q_j of the agent j.

Therefore, a novel idea is inspired to drive the motion center along l_{ji} instead of q_{ji} for the agent i, i.e.,

$$\mu_{ij} = \alpha(\|p_{ij}\|)l_{ji}.$$

Here l_{ji} is the projection of q_i on the normal axis of the agent j (see Fig. 1). In particular, l_{ji} can be calculated as follows. The vector $(p_j - q_i)$'s projection on $[\cos\theta_j, \sin\theta_j]^\top$ has the magnitude $|l_{ji}| = [\cos\theta_j, \sin\theta_j](p_j - q_i)$, therefore, $l_{ji} = [\cos\theta_j, \sin\theta_j]^\top|l_{ji}|$, that is,

$$l_{ji} = -k(\theta_j)\begin{bmatrix} x_{ij} - r_i\sin\theta_i \\ y_{ij} + r_i\cos\theta_i\end{bmatrix}/2,$$

$$k(\theta_j) := 2\begin{bmatrix} \cos^2\theta_j & \cos\theta_j\sin\theta_j \\ \sin\theta_j\cos\theta_j & \sin^2\theta_j\end{bmatrix}.$$

A technical lemma about the property of the function k is given below.

Lemma 3.1. *Assume $\theta_i(t), \theta_j(t) \in \mathbb{R}$ satisfy $\dot{\theta}_i = \ell_i\omega_o$ and $\dot{\theta}_j = \ell_j\omega_o$ for two positive integers ℓ_i and ℓ_j ($\ell_i \neq \ell_j$). Then,*

$$(1/T)\int_t^{t+T} k(\theta_i(\tau))k(\theta_j(\tau))d\tau = I, \quad T := 2\pi/\omega_o. \quad \square \tag{8}$$

Proof. Denote a matrix

$$A := \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} = (1/T)\int_t^{t+T} k(\theta_i(\tau))k(\theta_j(\tau))d\tau.$$

It suffices to show $A = I$. We note that, $\theta_i(\tau) = \ell_i\omega_o\tau + c_i$ and $\theta_j(\tau) = \ell_j\omega_o\tau + c_j$ for two constants c_i and c_j. Now, a direct calculation shows

$$\begin{aligned} A_{11} &= \frac{4}{T}\int_t^{t+T}[\cos^2\theta_i\cos^2\theta_j + \cos\theta_i\sin\theta_i\sin\theta_j\cos\theta_j]d\tau \\ &= 1 \end{aligned}$$

where we ignore the augment (τ) for θ_i and θ_j to save notations. Similarly, we have $A_{12} = 0, A_{21} = 0$ and $A_{22} = 1$. The proof is thus complete. $\square$

In particular, one can see that, the agent j's radius r_j is not necessary in the calculation of l_{ji}. But the self motion radius r_i is obviously available for the agent i. Moreover, we have

$$\begin{aligned} \mu_{ij} &= -\alpha(\|p_{ij}\|)k(\theta_j)\begin{bmatrix} x_{ij} - r_i\sin\theta_i \\ y_{ij} + r_i\cos\theta_i\end{bmatrix}/2 \\ &= -\alpha(\|p_{ij}\|)k(\theta_j)\begin{bmatrix} \xi_{ij} - r_j\sin\theta_j \\ \zeta_{ij} + r_j\cos\theta_j\end{bmatrix}/2 \\ &= -\alpha(\|p_{ij}\|)k(\theta_j)q_{ij}/2 \end{aligned}$$

by noting $k(\theta_j)[-\sin\theta_j, \cos\theta_j]^\top = 0$.

In summary, we have developed the following controller:

$$\begin{aligned} v_i &= r_i\omega_i - [\cos\theta_i \ \ \sin\theta_i] \\ &\quad \times \sum_{j\in\mathbb{N}, j\neq i}\alpha(\|p_{ij}\|)k(\theta_j)\begin{bmatrix} x_{ij} - r_i\sin\theta_i \\ y_{ij} + r_i\cos\theta_i\end{bmatrix}/2 \end{aligned} \tag{9}$$

or

$$\begin{aligned} v_i &= r_i\omega_i - [\cos\theta_i \ \ \sin\theta_i]R_i^{-1} \\ &\quad \times \sum_{j\in\mathbb{N}, j\neq i}\alpha(\|p_{ij}\|)R_i[\cos\theta_j \ \ \sin\theta_j]^\top[\cos\theta_j \ \ \sin\theta_j] \\ &\quad \times R_i^{-1}R_i(p_{ij} + r_i[-\sin\theta_i, \cos\theta_i]^\top). \end{aligned}$$

We note the following facts:

$$\begin{aligned} &\|p_{ij}\| = \|\wp_{ij}\|, \qquad [\cos\theta_i \ \ \sin\theta_i]R_i^{-1} = [\cos\vartheta_i \ \ \sin\vartheta_i], \\ &R_i[-\sin\theta_i, \cos\theta_i]^\top = [-\sin\vartheta_i, \cos\vartheta_i]^\top, \\ &R_i[\cos\theta_j \ \ \sin\theta_j]^\top[\cos\theta_j \ \ \sin\theta_j]R_i^{-1} \\ &\quad = [\cos(\theta_j - \varphi_i) \ \ \sin(\theta_j - \varphi_i)]^\top[\cos(\theta_j - \varphi_i) \ \ \sin(\theta_j - \varphi_i)] \\ &\quad = k(\vartheta_i - \theta_{ij}). \end{aligned}$$

As a result, the controller (9) can be rewritten as

$$\begin{aligned} v_i &= r_i\omega_i - [\cos\vartheta_i \ \ \sin\vartheta_i]\sum_{j\in\mathbb{N}, j\neq i}\alpha(\|\wp_{ij}\|) \\ &\quad \times k(\vartheta_i - \theta_{ij})(\wp_{ij} + r_i[-\sin\vartheta_i, \ \cos\vartheta_i]^\top)/2. \end{aligned} \tag{10}$$

Obviously, the controller (10) is of the form (2). From the above development, an interesting property of this controller is summarized as follows.

Theorem 3.1. *Consider the multi-agent system (1) under the decentralized controller (10) of the form (2). The closed-loop system is independent of the agents' individual reference frames, i.e., φ_i's.* $\square$

Proof. The proof easily follows the explicit controller development. In particular, with this controller (10), the closed-loop system becomes

$$\begin{aligned} \dot{q}_i &= [\cos\theta_i \ \ \sin\theta_i]^\top(v_i - r_i\omega_i) \\ &= [\cos\theta_i \ \ \sin\theta_i]^\top[\cos\theta_i \ \ \sin\theta_i]\mu_i, \end{aligned}$$

and hence

$$\dot{q}_i = -\sum_{j\in\mathbb{N}, j\neq i}k(\theta_i)k(\theta_j)\alpha(\|p_{ij}\|)q_{ij}/4.$$

The system can be put in a compact form

$$\dot{q} = -\mathcal{F}(q, \theta) \tag{11}$$

with

$$\begin{aligned} \mathcal{F}(q, \theta) &= \mathrm{col}(\mathcal{F}_1(q, \theta), \ldots, \mathcal{F}_n(q, \theta)), \mathcal{F}_i(q, \theta) \\ &= \sum_{j\in\mathbb{N}, j\neq i}k(\theta_i)k(\theta_j)\alpha(\|p_{ij}\|)q_{ij}/4. \end{aligned}$$

The closed-loop system (11) is obviously independent of φ_i's. $\square$

Now, the solution to the main controller synthesis problem is stated below.

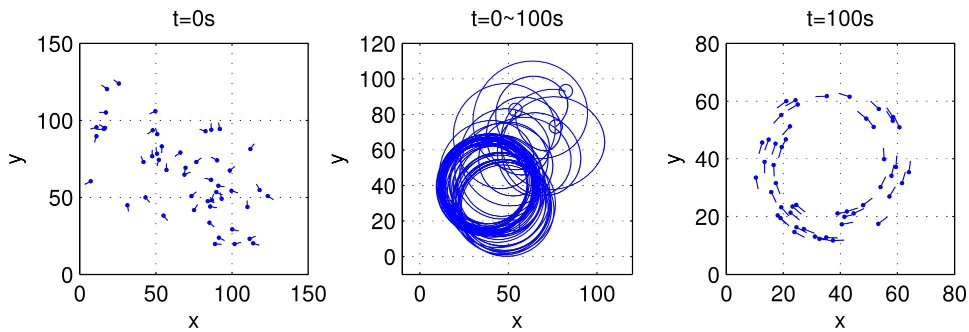

Fig. 2. Profile of a collective circular motion under the joint connectivity condition. Left: random initial distribution of the group; middle: trajectories of three agents (the initial potions are marked as ○); right: a collective circular motion is formed.

Theorem 3.2. *Consider the multi-agent system (1) under the decentralized controller (10) of the form (2). The closed-loop system is governed by approximate collective circular motion dynamics.* □

Proof. In the proof of Theorem 3.1, the closed-loop system is of the form (11). For the trajectory $q(t)$ of (11), we define a signal

$$\bar{q}(t) = (1/T)\int_t^{t+T} q(\tau)d\tau, \quad T = 2\pi/\omega_o.$$

It suffices to show that, for any $t \geq 0$, the properties (6) and (7) are satisfied.

First, we will show $\bar{q}(t) - q(t) = O(1/\omega_o)$. For any pair of agents i and j, if $\|p_{ij}\| > \rho$, we have $\alpha(\|p_{ij}\|) = 0$ since $\alpha \in \mathcal{S}(\rho)$; otherwise, we have $\|q_{ij}\| \leq \|p_{ij}\| + 2r \leq \rho + 2r$ is bounded and $\alpha(\|p_{ij}\|)$ is bounded. Recall that $r = \max_{i\in\mathbb{N}} r_i$. As a result, in any case, $\dot{q}$ are bounded, i.e., $\|\dot{q}\| < \sigma < \infty$, and hence

$$\|q(\tau) - q(t)\| = \left\|\int_t^{\tau} \dot{q}(s)ds\right\| \leq \sigma(\tau - t) \leq 2\pi\sigma/\omega_o,$$
$$\forall \tau \in [t, t+T],$$

which easily implies $\|\bar{q}(t) - q(t)\| \leq 2\pi\sigma/\omega_o$ or $\bar{q}(t) - q(t) = O(1/\omega_o)$.

Next, we will prove the Eq. (7). The function $f(q,\theta)$ is defined in (5). In particular, one has

$$f_i(q,\theta) = \sum_{j\in\mathbb{N}, j\neq i} f_{ij}(q,\theta), \quad f_{ij}(q,\theta) := \alpha(\|p_{ij}\|)q_{ij}/4.$$

From the definition of $f_{ij}(q,\theta)$, in particular, that α is Lipschitz continuous, it is easy to slightly modify the inequality (2.6) of Chen and Huang (2004) to obtain

$$\|f_{ij}(q^{(1)},\theta^{(1)}) - f_{ij}(q^{(2)},\theta^{(2)})\| \leq a_1\|q^{(1)} - q^{(2)}\| + a_2 r,$$
$$\forall q^{(1)}, \ \theta^{(1)}, q^{(2)}, \theta^{(2)}$$

for some positive constants a_1 and a_2. As a result, by noting $q(\tau) - q(t) = O(1/\omega_o)$ and $r = \max_{i\in\mathbb{N}}(v_{io}/\omega_{io}) = O(1/\omega_o)$, one has

$$f_{ij}(q(\tau),\theta(\tau)) - f_{ij}(q(t),\theta(t)) = O(1/\omega_o), \tag{12}$$
$$\forall \tau \in [t, t+T]. \tag{13}$$

Also, by noting $\bar{q}(t) - q(t) = O(1/\omega_o)$, one has

$$f_{ij}(\bar{q}(t),\theta(t)) - f_{ij}(q(t),\theta(t)) = O(1/\omega_o). \tag{14}$$

Then, we are ready to have the following calculation:

$$\begin{aligned}\dot{\bar{q}}_i(t) &= \frac{1}{T}\frac{d}{dt}\int_t^{t+T} q_i(\tau)d\tau = \frac{1}{T}\int_t^{t+T}\frac{d}{d\tau}q_i(\tau)d\tau \\ &= -\frac{1}{T}\int_t^{t+T}\sum_{j\in\mathbb{N}, j\neq i} k(\theta_i(\tau))k(\theta_j(\tau))f_{ij}(q(\tau),\theta(\tau))d\tau.\end{aligned}$$

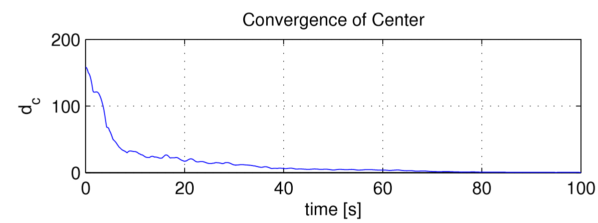

Fig. 3. Convergence performance of the circular motion centers.

Using (13) and (14), one has

$$\begin{aligned}\dot{\bar{q}}_i(t) = &-\frac{1}{T}\sum_{j\in\mathbb{N}, j\neq i}\int_t^{t+T} k(\theta_i(\tau))k(\theta_j(\tau))d\tau(f_{ij}(\bar{q}(t),\theta(t)) \\ &+ O(1/\omega_o)) = -f_i(\bar{q}(t),\theta(t)) + O(1/\omega_o).\end{aligned}$$

In the last equation, we note

$$(1/T)\int_t^{t+T} k(\theta_i(\tau))k(\theta_j(\tau))d\tau = I.$$

The Eq. (7) is thus proved. □

4. Numerical simulation and experiments

We consider a model composed of (1) and (10) with $\alpha(x) = 0.05(1 - x/\rho)$ for $0 \leq x \leq \rho$. The nominal individuals have different linear speeds and rotational speeds. To address the joint connectivity condition in Proposition 2.1, we restrain the group starting within a square space of $L \times L$ with $L = 150$. Since all individuals start from a bounded space $L \times L$, we assume that the union of the proximity nets over sequential time-intervals $[t_i, t_{i+1})$, $i = 1, \ldots, \infty$, is connected, and the condition of Proposition 2.1 is thus satisfied. To describe how the circular motion center of each agent asymptotically converges, we define the maximal value of the distances between two agents' circular motion centers as follows:

$$d_c(t) = \max\{\|q_{ij}(t)\|, i, j \in \mathbb{N}\}.$$

Once the circular motion behavior is achieved, all centers converge to one single point, i.e., $\lim_{t\to\infty} d_c(t) = 0$. With $n = 50$ and $15 \leq r_i \leq 25$, $i \in \mathbb{N}$, the simulation results are given in Fig. 2. The left graph shows the initial distribution of the group of agents at $t = 0$ s. A collective circular motion is formed as expected by Proposition 2.1 when the distribution is shown in the right graph at $t = 100$ s. The motion trajectories of three agents (not all 50 agents) are plotted in the middle graph. The trajectories eventually converge to circles with a common center. The convergence performance of $\lim_{t\to\infty} d_c(t) = 0$ is demonstrated in Fig. 3.

An experimental multi-robots system is used to examine the practical applicability of the control algorithm proposed in this

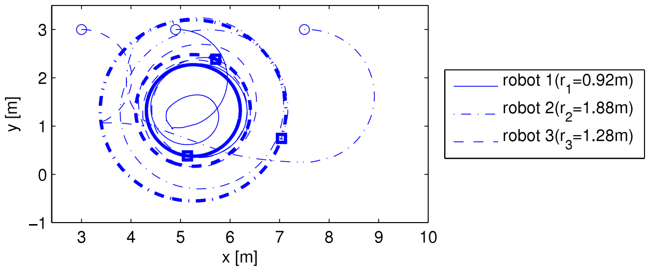

Fig. 4. Trajectories of three robots. The initial positions are marked as ○ and the final positions are marked as □. The trajectories are highlighted in bold when a collective circular motion is achieved.

paper. The multi-robots system is composed of three Pioneer 2-DXe robots of Adept MobileRobots (Laffary, 2002). The detailed description of the experimental platform in hardware and software can be found in Zhang, Chen, Yan, and Yu (2012) where the algorithm for homogeneous agents is studied. In the experiments, the complete moving trajectories of all robots were recorded in an odometer log and plotted in Fig. 4. It was observed that, all the three robots initially moved along their own circular trajectories, and the motion trajectories eventually converged to circles with a common center after several times of interactions. It is worth mentioning that, an obstacle avoidance algorithm was added in the practical controller implementation for the significant size of robots. The sharp turns in the trajectories correspond to the activation of the obstacle avoidance algorithm. A more detailed explanation of the algorithm can be found in Zhang et al. (2012).

5. Conclusion

In this paper, a completely decentralized control algorithm has been proposed for a group of autonomous multi-agents to form a class of collective circular behavior without a common rotational radius or a common reference frame. The approach has been demonstrated by a 2D torus simulation and experiments with guaranteed convergence of circular motion centers. In the resulting torus pattern, all agents move in circles around a common center, but with different radii. Such a torus pattern is more similar to many real phenomena of natural systems than the existing torus pattern with a same rotational radius and may have more potential applications.

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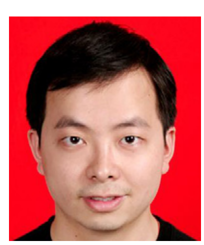
Zhiyong Chen received the B.E. degree from the University of Science and Technology of China, and the M.Phil. and Ph.D. degrees from the Chinese University of Hong Kong, in 2000, 2002, and 2005 respectively. He worked as a Research Associate at the University of Virginia during 2005–2006. He is now a Senior Lecturer at the University of Newcastle, Australia. His research interests include nonlinear systems and control, biological systems, and multi-agent systems. He is an Associate Editor of IEEE Transactions on Automatic Control.



Hai-Tao Zhang received the B.E. and Ph.D. degrees from the University of Science and Technology of China, in 2000 and 2005 respectively. He worked as a Postdoctoral Researcher at the University of Cambridge, during January–December 2007. Since 2005, he has been with Huazhong University of Science and Technology, where he was an Associate Professor from 2005 to 2010 and afterward a Professor from late 2010 till now. His research interests include multi-agent systems control, model predictive control and nano-manufacturing control.