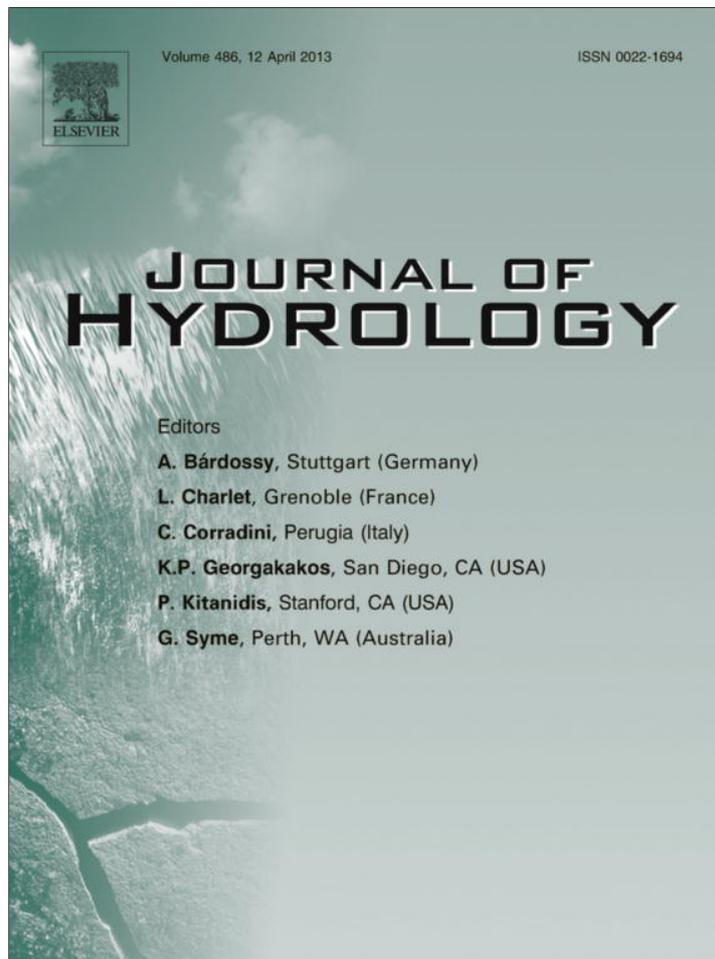


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## Estimation of spatial soil moisture averages in a large gully of the Loess Plateau of China through statistical and modeling solutions

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## SUMMARY

Characterizing root-zone soil moisture patterns in large gullies is challenging as relevant datasets are scarce and difficult to collect. Therefore, we explored several statistical and modeling approaches, mainly focusing on time stability analysis, for estimating spatial soil moisture averages from point observations and precipitation time series, using 3-year root-zone (0–20, 20–40, 40–60 and 60–80 cm) soil moisture datasets for a large gully in the Loess Plateau, China. We also developed a new metric, the root mean square error (RMSE) of estimated mean soil moisture, to identify time-stable locations. The time stability analysis revealed that different time-stable locations were identified at various depths. These locations were shown to be temporally robust, by cross-validation, and more likely to be located in ridges than in pipes or plane surfaces. However, we found that MRD (mean relative difference) operators, used to predict spatial soil moisture averages by applying a constant offset, could not be transferred across root zone layers for most time-stable locations. Random combination analysis revealed that at most four randomly selected locations were needed for accurate estimation of mean soil moisture time series. Finally, a simple empirical model was developed to predict root-zone soil moisture dynamics in large gullies from precipitation time series. The results showed that the model reproduced root-zone soil moisture well in dry seasons, whereas relatively large estimation error was observed during wet seasons. This implies that only precipitation observations might be not enough to accurately predict root-zone soil moisture dynamics in large gullies, and time series of soil moisture loss coefficient should be modeled and included.

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### 1. Introduction

Gullies occur globally in areas with crusting soils, such as loess (European belt, Chinese Loess Plateau, North America), sandy soils (Sahelian zone, north-east Thailand) and dispersive soils prone to piping and tunneling following serious vegetation disturbance, e.g. overgrazing (Valentin et al., 2005). Their occurrence can result in serious soil erosion and land degradation, thus they are responsible for most deposition of sediments in downstream pools in these areas (Li et al., 2003; Melliger and Niemann, 2010; Valentin et al., 2005). Their formation also impairs the ecology of adjacent uplands, accelerating soil desiccation (Huo et al., 2008; Zheng et al., 2006) and reducing both their grazing value and agricultural potential (Avni, 2005; Krause et al., 2003).

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Soil moisture is a key hydrological and ecological variable in land surface systems. In gullies, root-zone soil moisture may control hydrological and ecological processes in the following ways. First, it affects the generation of surface runoff (Ludwig et al., 2005), the driving force of soil erosion. Second, it affects the soil shear strength, because as soil moisture approaches saturation the shear strength decreases and soils become prone to erosion (Collins and Bras, 2008). Third, it may affect rates of gully incision and reshape gully topography (Melliger and Niemann, 2010), which may interactively change soil moisture distributions. Fourth, soil moisture influences the streamflow along gully bottoms, and thus outlet discharges (Kirchner, 2009). Finally, in arid and semi-arid areas, soil moisture has major effects on vegetation structure and organization (Rodriguez-Iturbe et al., 1999); under severe water stress vegetation may die, leading to the acceleration of soil erosion. Various studies have indicated that soil moisture patterns in gullies differ substantially from those in uplands (Gao et al., 2011a; Melliger and Niemann, 2010; Van den Elsen et al., 2003). Furthermore, the hydrological connectivity between gullies and

uplands may be weak because vertical hydrological fluxes are much stronger than lateral fluxes in gullies (Grayson and Western, 2001). Thus, gullies should be considered separately from hill-slopes (uplands) in attempts to elucidate soil moisture variations and the associated hydrological and ecological processes in gullied areas.

In situ soil moisture measurement techniques have greatly improved recently following the introduction of instruments such as neutron thermalization probes, electromagnetic sensors and heat pulse sensors. However, knowledge of spatial–temporal soil moisture patterns in gullies is still very limited, due to the difficulty of sampling soil moisture in them and the associated expensive costs (Gao et al., 2011a), which hinders elucidation of hydrological and ecological processes in areas where they are prevalent. Satellite-based sensors may provide soil moisture products at fine temporal resolution (1–3 days), but their spatial resolution is relatively low (~25–50 km) (Crow et al., 2012). Furthermore, they are sensitive to land surface features (topography and vegetation) (Pathe et al., 2009). Hence, ground calibration and validation are required to interpret satellite soil moisture maps of gullies. As such, intensive in situ measurements in space and time (expensive and laborious) should be conducted to obtain accurate soil moisture information (Zhou et al., 2007). In order to save economic and labor costs, in situ soil moisture sampling locations should be optimized. Therefore, upscaling methods or modeling approaches are needed to obtain long-term soil moisture time series in gullies for evaluating soil wetness conditions and calibrating remote sensing soil moisture products (Brocca et al., 2010; Zhou et al., 2007).

An effective method for upscaling spatial soil moisture means is time stability analysis, introduced by Vachaud et al. (1985) to characterize time-invariant associations between spatial locations and classical statistical parametric values. Time stability is also known as temporal stability (Vachaud et al., 1985), temporal persistence (Kachanoski and de Jong, 1988), and rank stability (Chen, 2006). The concept has been widely used to characterize the temporal persistence of spatial patterns of soil moisture fields and identify locations that accurately represent spatial averages for various purposes, including: ground calibration of soil moisture maps obtained by remote sensing (e.g., Brocca et al., 2010, 2012; Choi and Jacobs, 2007; Cosh et al., 2004; De Lannoy et al., 2007; Grayson and Western, 1998; Heathman et al., 2012; Jacobs et al., 2004, 2010; Joshi et al., 2011; Mohanty and Skaggs, 2001; Starks et al., 2006); spatial soil water storage estimation (e.g., Gao et al., 2011b; Gao and Shao, 2012; Hu et al., 2010a) and inferring missing values (e.g., Dumedah and Coulibaly, 2011; Guber et al., 2008; Pachepsky et al., 2005). However, time stability features of soil moisture have never been tested in large gullies. Furthermore, although time stability was considered in the cited studies, few attempted to validate identified time-stable locations or address effects of temporal variations in vegetation type and precipitation (Han et al., 2012), with the exception of Hu et al. (2010a), Jacobs et al. (2010) and Martínez-Fernández and Ceballos (2005). Moreover, the time stability of soil moisture is closely linked to other environmental factors, such as soil texture (e.g., Jacobs et al., 2004; Mohanty and Skaggs, 2001), topography (e.g., Gao et al., 2011b; Grayson and Western, 1998), and vegetation features (Jacobs et al., 2010; Joshi et al., 2011). For instance, both Joshi et al. (2011) and Mohanty and Skaggs (2001) found that time stability was generally higher in sandy loam fields than in silt loam fields in the South Great Plain (SGP) region, and higher in gently rolling fields than in areas with flat topography. Jacobs et al. (2004) showed that locations with mild slopes had higher time stability than hilltops and steep slopes, and that soil parameters were insufficient to predict temporally stable locations in Walnut Creek watershed, Iowa. Joshi et al. (2011) also found that time stability was highest on hilltops in Iowa.

A prerequisite for time stability analysis is intensive soil moisture sampling to identify robust time-stable locations (Teuling et al., 2006). Thus, this method is not applicable in ungauged areas. When previous soil moisture time series are not available, an alternative method is random combination analysis, introduced by Wang et al. (2008). Brocca et al. (2010, 2012) found that only a few locations are required to reliably estimate the mean soil moisture for an area. Accordingly, combining time stability analysis and random combination analysis may provide all the information required for optimizing in situ soil moisture networks (Brocca et al., 2012). However, few studies have combined these methods to analyze the spatiotemporal variability of soil moisture.

The main driver of changes in soil moisture is precipitation (Entekhabi and Rodriguez-Iturbe, 1994), which (unlike soil moisture) is usually recorded routinely at weather stations. Therefore, several attempts have been made to develop models linking soil moisture to precipitation (Entekhabi and Rodriguez-Iturbe, 1994; Pan et al., 2003; Yoo et al., 1998). Entekhabi and Rodriguez-Iturbe (1994) introduced a stochastic partial differential equation to characterize the spatiotemporal variability of soil moisture. Pan et al. (2003) modified this partial differential equation, by dropping a diffusion term, in a successful attempt to obtain a simplified model capable of predicting surface soil moisture (0–5 cm) from precipitation observations. The key to reliable application of this model is determination of appropriate time window sizes, which may vary substantially among different areas. Moreover, this model also requires inputs of land surface features and soil properties (e.g., infiltration rates). Thus, despite its success there is a need for a simpler model for estimating root-zone soil moisture patterns in large gullies from precipitation observations, without sampling and determining soil properties, which is difficult in complex gully topography.

On the Loess Plateau, gullies cover approximately 40% of the total area, at densities of 1.5–4.0 km km<sup>-2</sup> (Zheng et al., 2006), rising to 50–60% and densities of 3–8 km km<sup>-2</sup> in hilly parts (Huang and Ren, 2006). However, despite the very large area they cover in the Loess Plateau, knowledge of spatiotemporal soil moisture variations in the gullies is very limited. Thus, the objectives of the presented study were to develop and evaluate methodologies for estimating spatial root-zone soil moisture averages in large gullies, by analyzing its time stability at various locations in a selected gully and then estimating spatial averages of soil moisture from point observations, using two approaches. The first approach, applicable when previous time series of soil moisture data are available, involves time stability analysis and use of a new metric introduced for identifying time-stable locations to upscale point observations. The second approach, applicable when previous time series are not available, involves use of a random combination method. In particular, a simple empirical model was developed for estimating spatial mean root-zone (0–80 cm) soil moisture in gullies during the growing season. Although the calibration and validation of remote sensing soil moisture products in large gullies are not the main purpose of this investigation, this study may facilitate the calibration and validation work for the Loess Plateau region, where little relevant work has been done. This is becoming increasingly important with the increasing availability of coarse-resolution satellite-based soil moisture maps (e.g., Advanced Microwave Scanning Radiometer, AMSR; Advanced SCATterometer, ASCAT; and Soil Moisture and Ocean Salinity, SMOS, maps).

## 2. Data description

### 2.1. Study site

The study field is located at the Yuanzegou catchment (37°15'N, 118°18'E) (Fig. 1) which is a typical gully catchment in the hilly

areas of the Loess Plateau and gullies occupies approximately 53% (0.31 km<sup>2</sup>) of the total area of the catchment (0.58 km<sup>2</sup>). This area has a semiarid continental climate with (based on data for 1956–2006): mean annual precipitation of 505 mm, 70% of which falls during late summer and early autumn (August, September and October); a mean annual temperature of 8.6 °C, with mean monthly temperatures ranging from –6.5 °C in January to 22.8 °C in July; 157 frost-free days and 2720 h of sunshine on average each year (Weather Bureau of Qingjian county, Shaanxi province). The elevation of the Yuanzegou catchment ranges from 865 to 1105 m. The main gully stretches from south to north, with prevalent steep slopes of 35–90°.

The gully sidewalls are the main part of the gullies (very typical in hilly areas of the Loess Plateau) and they are weakly disturbed by human activity. Soils in gullies are primarily composed of loess with texture of fine silt and silt loam. The basic soil hydraulic properties in gullies refer to Gao et al. (2011a). Soil thickness ranges from less than 0.2 m on the gully floor to more than 15 m at gully edges. The soils are primarily vegetated with perennial grasses, including *Artemisia gmelinii*, *Bothriochloa ischemum* and *Lespedeza davurica*. According to Gao et al. (2011a), the root zone layer was defined as the 0–80 cm because the majority (>90%) of root mass for most of the plants in this site was within this depth.

## 2.2. Soil moisture collection

Because most of gully floors are consisted of exposed bedrock, only soil moisture over gully sidewalls was sampled. Three transects (A, B and C), traversing the gully sidewalls to represent the range of topography with the lengths of approximately 50, 80 and 50 m long, respectively, were established to collect soil moisture (Fig. 1). In general, 9 locations were sampled along transect B, and 5 along transects A and C; there was a distance of approximately 10–15 m between sampling points. According to Gao et al. (2011a), the special topographic positions in gully sidewalls includes ridges, pipes, plane surfaces, and cliffs. These sampling locations covered the different topographic positions (except for cliffs where it is impossible to conduct soil moisture sampling), with at least five locations for each position. It seems that the total number of locations may be not sufficient to obtain spatial statistical results. However, the sampling locations here cover different micro-topography in gullies. Therefore, this study assumes that spatially distributed sampling locations could provide soil moisture characteristics in gullies.

From September 3 2009 to September 19 2011, soil moisture in 0–80 cm was collected with an interval of 20 cm at each sampling location and a total of 41 sampling days were conducted. The total

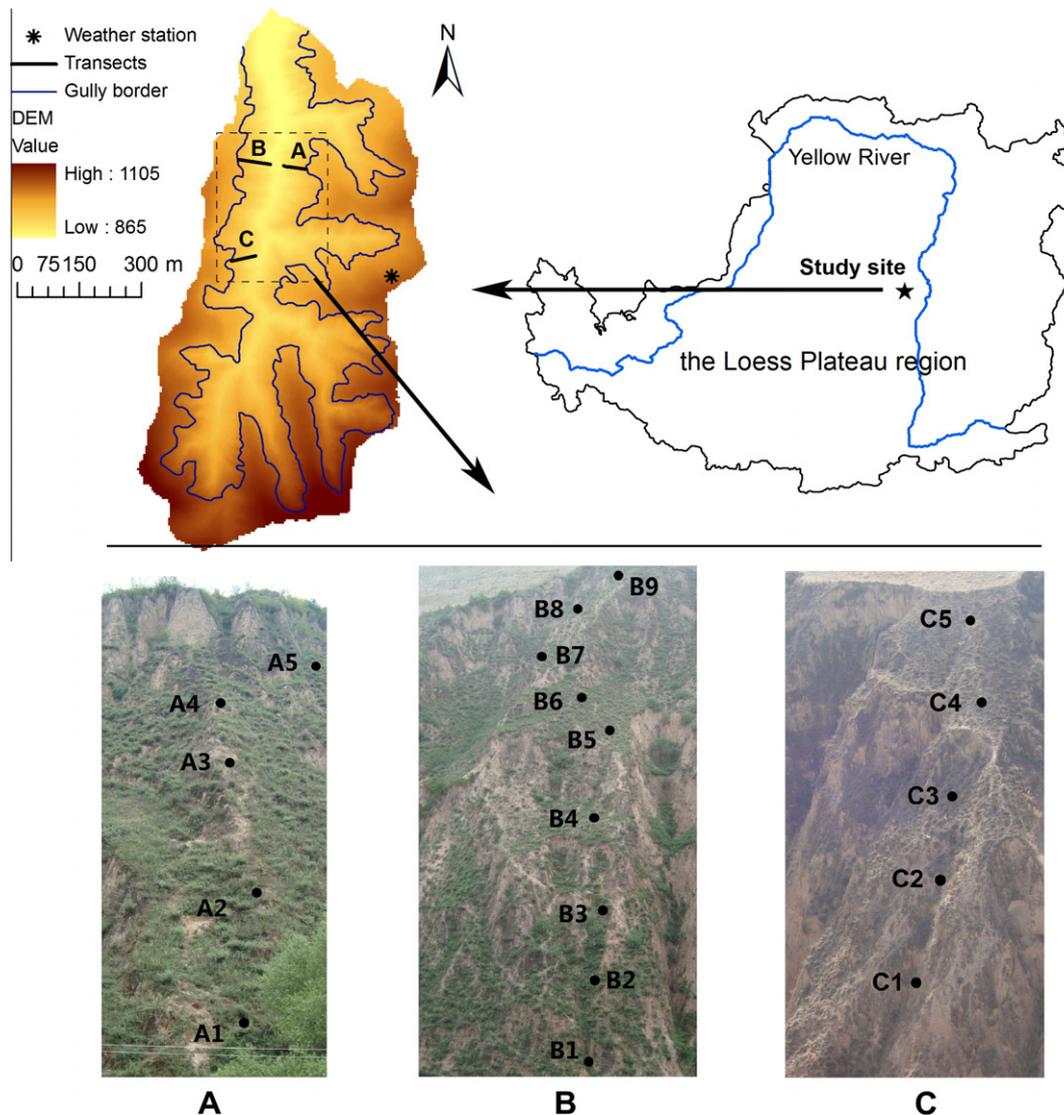


Fig. 1. Study site and sampling transects in gullies.

precipitation for the year of 2009, 2010 and 2011 was 654.3, 426.8 and 499.6 mm, respectively. A portable Time Domain Reflectometry (TDR) system, TRIME-PICO IPH/T3 (IMKO, Ettlingen, Germany) was used to collect soil moisture. The TDR system consists of a TRIME-IPH probe, a TRIME-DataPilot datalogger and fiberglass access tubes ( $\phi = 40$  mm). A hand auger ( $\phi = 45$  mm) was used to install fiberglass access tubes instead of the original accessories, which are difficult to operate over gully sidewalls. To facilitate installation of the tubes, the relief at some sampling points was disturbed slightly. The space between the tube and soil was filled with a mixture of the soil removed by the hand auger and water (Gao et al., 2011a). For each sampling event, three repeats were made at each depth and soil moisture was sampled within 4 min at each sampling location. The soil moisture measurements for all locations were taken within 2 h to diminish the temporal variation of soil moisture as much as possible. Before soil moisture sampling, we conducted a local gravimetric calibration for this TDR system. The detailed calibration process is described in Gao et al. (2011a), using the following calibration equation:

$$y = 0.9471x - 4.3796, \quad R^2 = 0.904, \quad \text{RMSE} = 2.68\% \quad (1)$$

where  $x$  is the TDR-derived moisture value (% v/v);  $y$  is the volumetric moisture content transformed from the gravimetric moisture content by multiplying it by the corresponding bulk density (% v/v).

### 2.3. Land surface features

Disturbed soil samples at the surface depth (0–20 cm) were collected by a hand auger ( $\phi = 40$  mm) from close to each soil moisture sampling location in order to determine the soil particle size distribution; and this was achieved using the MS2000 particle size analyzer (Malvern Instrument, Malvern, Britain). A geological compass was used to determine the slope angle and the aspect, and a portable global positioning system (GPS) (GPS 72H™, Garmin, USA) was used to determine the elevation for each sampling location in gullies. The vegetation cover was measured by using a cross-hair point frame with a size of 50 cm \* 50 cm and 100 grids, and the number of the plants was counted within the frame. The detailed information for these environmental factors was shown in Table 1.

## 3. Methods

In this study, time stability analysis was the primary method for estimating mean soil moisture in gullies. Random combination analysis and an empirical model were also used for analyses in order to provide more comprehensive information for mean soil moisture estimation in large gullies, considering different cases (previous soil moisture dataset available or not).

### 3.1. Time stability analysis

#### 3.1.1. Method description

Following Vachaud et al. (1985), for a given depth  $k$  and sampling day  $j$ , the relative difference (RD) for sampling location  $i$  with respect to spatial mean soil moisture at the same depth ( $\bar{\theta}_{jk}$ ) is defined as:

$$\delta_{ijk} = \frac{\theta_{ijk} - \bar{\theta}_{jk}}{\bar{\theta}_{jk}} \quad (2)$$

The temporal mean relative difference (MRD) and its standard deviation (SDRD) during the study period are calculated as:

$$\bar{\delta}_{ik} = \frac{1}{T} \sum_{j=1}^T \delta_{ijk} \quad (3)$$

and

$$\sigma_{ik}(\delta) = \sqrt{\frac{1}{T-1} \sum_{j=1}^T (\delta_{ijk} - \bar{\delta}_{ik})^2} \quad (4)$$

where  $T$  is the total number of sampling days over the study period. Positive and negative MRD values signify that the considered location is wetter and drier, respectively than the spatial mean soil moisture. The SDRD can be used to discern time-stable locations; the lower this parameter, the more temporally stable the location. However, since the main purpose of time stability analysis is to estimate spatial mean soil moisture from time-stable locations, a shortcoming of using SDRD is that it does not directly reflect estimation errors (Hu et al., 2010a).

According to Grayson and Western (1998), time-stable locations with non-zero relative differences can be used to represent

**Table 1**  
Overall information of environmental variables for sampling locations in the study site.

ID	Sand content <sup>a</sup> (%)	Silt content <sup>a</sup> (%)	Clay content <sup>a</sup> (%)	SOC <sup>b</sup> (g kg <sup>-1</sup> )	STN <sup>c</sup> (g kg <sup>-1</sup> )	Slope (%)	Cos (aspect)	Relative elevation (m)	Position	Plant cover (%)	Plant density (plants m <sup>-2</sup> )
A1	15.7	63.8	20.5	3.50	0.311	64.9	0.14	21	Plane surface	54	22
A2	19.7	66.9	13.4	2.77	0.292	70.0	0.05	27	Plane surface	45	17
A3	23.2	62.1	14.7	1.67	0.178	90.0	0.10	41	Ridge	21	2
A4	23.4	61.4	15.2	3.61	0.303	83.9	0.10	51	Ridge	34	6
A5	22.5	64.6	12.9	2.33	0.246	55.4	0.26	62	Plane surface	59	18
B1	12.4	68.5	19.1	5.42	0.293	160.0	0.09	22	Pipe	52	9
B2	12.1	66.8	21.1	4.04	0.444	142.8	-0.09	33	Pipe	26	4
B3	17.2	66.3	16.5	3.59	0.329	103.6	0.00	42	Pipe	18	2
B4	22.5	66.0	11.5	2.56	0.223	32.5	-0.09	51	Plane surface	47	10
B5	15.3	65.8	18.9	3.21	0.179	64.9	0.42	61	Plane surface	49	9
B6	14.4	66.2	19.4	5.19	0.180	72.7	0.50	69	Plane surface	60	14
B7	24.1	65.7	10.2	1.98	0.336	72.7	-0.33	78	Ridge	29	5
B8	22.8	64.5	12.7	2.93	0.362	70.0	-0.42	90	Ridge	25	3
B9	23.6	63.3	13.1	3.24	0.389	60.1	-0.39	99	Ridge	30	5
C1	15.9	64.5	19.6	1.99	0.190	180.4	-0.53	46	Pipe	10	1
C2	16.8	67.2	16.0	2.43	0.243	103.6	-0.47	55	Pipe	24	4
C3	14.8	66.4	18.8	2.78	0.255	51.0	0.29	64	Plane surface	39	12
C4	17.6	66.5	15.9	3.59	0.317	60.1	0.37	72	Plane surface	61	18
C5	19.1	66.2	14.7	3.18	0.392	67.5	0.37	78	Plane surface	75	20

<sup>a</sup> Sand content: 0.02–2 mm; silt content: 0.002–0.2 mm; clay content: <0.002 mm.

<sup>b</sup> SOC: soil organic carbon.

<sup>c</sup> STN: soil total nitrogen.

spatial mean soil moisture, provided the offset (RD) between the non-zero time-stable locations and the mean value is known. Rearranging Eq. (2), the spatial mean soil moisture at depth  $k$  can be expressed as:

$$\bar{\theta}_{jk} = \frac{\theta_{ijk}}{1 + \delta_{ijk}} \quad (5)$$

Assuming a constant coefficient equaling the MRD, the spatial mean soil moisture can then be indirectly estimated as:

$$\tilde{\theta}_{jk} = \frac{\theta_{ijk}}{1 + \bar{\delta}_{ik}} \quad (6)$$

Thus, the root mean square error (RMSE) of the estimated spatial mean soil moisture, applying a constant offset, was calculated as follows:

$$\text{RMSE}_i(\tilde{\theta}) = \sqrt{\frac{1}{T} \sum_{j=1}^T (\tilde{\theta}_{jk} - \bar{\theta}_{jk})^2} \quad (7)$$

Grayson and Western (1998) further stated that the offset coefficients (MRD) for time-stable locations are “constants” that are independent of the time of year (or average wetness). However, this has been questioned by Heathman et al. (2012), because several studies have shown that the linear offset coefficients calculated in this manner are not necessarily transferable in time and space, as they may be affected by variations in climate, temporal sampling or vegetation. Although we agree with the arguments of Heathman et al. (2012), offset coefficients (MRD) for time-stable locations (with low SDRD), should theoretically be less sensitive to variations in the time of year (and wetness) than those for other locations. Therefore, the RMSE parameter used in this study is closely related to the time stability. For time-stable locations, the estimated spatial mean soil moisture ( $\tilde{\theta}_{jk}$ ) should be more likely to approximate the true spatial mean soil moisture ( $\bar{\theta}_{jk}$ ), and the RMSE should be lower than corresponding parameters for other locations. In this sense, the RMSE applied here can be used to identify time-stable locations and it directly reflects the estimation error when a constant offset coefficient is applied. Generally, the lower the RMSE, the more temporally stable a location is. In this study, we used RMSE to identify time-stable locations, and Eq. (6) was then employed to estimate spatial averages of root-zone soil moisture in gullies from measurements of time-stable locations. In addition, Eq. (6) has also been used as an observation operator (Han et al., 2012) for spatial mean soil moisture estimation. It was termed MRD operator here.

Han et al. (2012) argued that time-stable locations may differ in different periods, partly at least because of limitations in sampling periods. Therefore, to identify robust time-stable locations, the whole data sets (covering 41 sampling days) were used for time-stability analysis, and leave-one-out cross-validation was then employed to test their robustness, as described below.

### 3.1.2. Cross-validation

Cross-validation was conducted, according to Jacobs et al. (2010), as follows. Each of the 40 datasets obtained by omitting soil moisture data for one of the 41 sampling days ( $T$ ) was used to identify time-stable locations, for each depth, and the remaining data set in each case (the  $j$ th data set, where  $j = 1, 2, \dots, T$ ) was used for validation. First, transform the soil moisture for time-stable locations derived from the 40 subsets to spatial averages of gullies through Eq. (6); then average the transformed spatial means according to Eq. (9) and compared to the mean value of the remaining data set, again for each depth. Here, the first five time-stable locations (based on the 40 subsets) were identified for each sampling day, i.e.,  $N = 5$ . The accuracy of the averages was then evaluated by calculating the root mean square

error (RMSE). The equation for calculating RMSE at the  $j$ th sampling day and depth  $k$  was:

$$\text{RMSE}_{jk} = \sqrt{\frac{1}{N} \sum_{s=1}^N (\tilde{\theta}_{jk}(s) - \bar{\theta}_{jk})^2} \quad (8)$$

$$\tilde{\theta}_{jk}(s) = \frac{1}{s} \sum_{s=1}^s \tilde{\theta}_{sjk} \quad (9)$$

where  $s$  is the number of time-stable locations;  $\tilde{\theta}_{sjk}$  is the spatial mean soil moisture of gullies estimated from the  $s$ th time-stable location according to Eq. (6);  $\tilde{\theta}_{jk}(s)$  is the estimated spatial mean soil moisture of gullies based on time-stable locations.

### 3.1.3. Statistical metrics

The correlation coefficient ( $R$ ), root mean square error (RMSE), and mean bias error (MBE) were calculated as measures of the goodness-of-fit between observed ( $E_{jk}$ ) and estimated ( $O_{jk}$ ) mean soil moisture contents, using the following equations:

$$R_k = \frac{\sum_{j=1}^T (E_{jk} - \bar{E}_{jk})(O_{jk} - \bar{O}_{jk})}{\sqrt{\sum_{j=1}^T (E_{jk} - \bar{E}_{jk})^2 \sum_{j=1}^T (O_{jk} - \bar{O}_{jk})^2}} \quad (10)$$

$$\text{RMSE}_k = \sqrt{\frac{1}{T} \sum_{j=1}^T (E_{kj} - O_{kj})^2} \quad (11)$$

$$\text{MBE}_k = \frac{1}{T} \sum_{j=1}^T (E_{kj} - O_{kj}) \quad (12)$$

where  $T$  is the total number of sampling days, while  $\bar{E}_{jk}$  and  $\bar{O}_{jk}$  are the temporal means of  $E_{jk}$  and  $O_{jk}$ , respectively.

### 3.2. Random combination method

If previous soil moisture time series are not available, a random combination method was used to obtain the number of locations (randomly selected) needed to estimate spatial mean soil moisture within a given accuracy (Brocca et al., 2010, 2012; Wang et al., 2008). In particular, the following steps are included for this method (Brocca et al., 2010, 2012):

- (1) For a given depth  $k$ , randomly select  $M$  point measurements ( $1 \leq M \leq N$ ) from the available  $N$  observations with  $R$  replicates.
- (2) For each replicate, the time series of spatial mean soil moisture is calculated, and thus a total of  $R$  soil moisture time series are obtained.
- (3) These time series are compared statistically with the one based on all the  $N$  sampling locations (termed as benchmark time series). Two metrics, the determination coefficient ( $R^2$ ) and the root mean square error, are used for this comparison.
- (4) The mean and standard deviation of the two statistical moments are assessed.
- (5) Steps (1)–(4) are repeated for  $M$  ranging from 1 to  $N$ .

Here we set a criterion to select the minimum number of randomly selected locations, that is the determination coefficient  $>98\%$  and the root mean square error  $<1\%$ .

### 3.3. Estimating spatial mean values from precipitation time series

We hypothesized that precipitation and evapotranspiration are the main factors controlling root-zone soil moisture dynamics at

the study site because the groundwater table in the Loess Plateau is usually deeper than 50 m (Yang and Shao, 2000). Therefore, it should be possible to estimate root-zone soil moisture from precipitation observations.

In this part of the study we developed a simple empirical model to estimate spatial mean soil moisture in gullies from precipitation time series. Generally, the relationship between root-zone soil moisture content and daily mean evapotranspiration is strong, and linear under water stress conditions (Basara and Crawford, 2002; Shang et al., 2000). According to Shang et al. (2000), the relationship over a period without rainfall can be expressed as:

$$\frac{d\omega}{dt} = -k\omega \quad (13)$$

where  $\omega$  is the spatial mean root-zone soil moisture in a given gully (mm); and  $k$  is the soil moisture loss coefficient ( $d^{-1}$ ), the minus symbol denoting the reduction of soil moisture. Integrating Eq. (13) for time interval  $[t_i, t_j]$  gives:

$$\omega_{t_j} = \omega_{t_i} \exp(-k_i \Delta t_i) \quad (14)$$

and

$$k_i = \frac{\ln \omega_{t_i} - \ln \omega_{t_j}}{t_j - t_i} \quad (15)$$

Here,  $\omega_{t_j}$  is the spatial mean root-zone soil moisture at time  $t_j$ ;  $\omega_{t_i}$  is the spatial mean root-zone soil moisture at time  $t_i$ ; and  $k_i$  is the loss coefficient for time interval  $[t_i, t_j]$ . Including a precipitation term in Eq. (14) gives:

$$\omega_{t_j} = \omega_{t_i} \exp(-k_i \Delta t_i) + P_i \quad (16)$$

If the time interval is 1 day, i.e.,  $\Delta t_i = 1$  d, then,

$$\omega_{t_j} = \omega_{t_i} \exp(-k_i) + P_i \quad (17)$$

If the initial time is  $t_1$ , and the initial spatial mean root-zone soil moisture is  $\omega_{t_1}$ , then,

$$\omega_{t_2} = \omega_{t_1} \exp(-k_1) + P_1 \quad (18)$$

$$\omega_{t_3} = \omega_{t_1} \exp[-(k_1 + k_2)] + P_1 \exp(-k_2) + P_2 \quad (19)$$

Applying the above procedure, we can now derive the following equation,

$$\omega_{t_i} = \omega_{t_1} \exp\left(-\sum_{i=1}^{i-1} k_i\right) + P_1 \exp\left(-\sum_{i=2}^{i-1} k_i\right) + \dots + P_{i-1} \quad (20)$$

where  $P_1$  is the precipitation during time interval  $\Delta t_1$ , and  $P_{i-1}$  is the precipitation during time interval  $\Delta t_{i-1}$ . Using the temporal mean ( $\bar{k} = \frac{1}{N} \sum_{i=1}^N k_i$ , where  $N$  is the total number of time intervals for a given period) of  $k_i$  to substitute  $k_i$  at each time interval, one has,

$$\omega_{t_i} = \omega_{t_1} \exp[-(i-1)\bar{k}] + P_1 \exp[-(i-2)\bar{k}] + \dots + P_{i-1} \quad (21)$$

In Eq. (21), only precipitation is temporally variable, and in this study it was used to predict spatial mean root-zone soil moisture in the studied gully from precipitation time series recorded at a nearby weather station (~200 m from the edge of the gully). The estimation error was also calculated.

## 4. Results and discussion

### 4.1. Statistical description

The soil moisture datasets for analyses were derived from different topographic positions (pipe, plane surface and ridge) of the large gully and spanned dry, medium and wet seasons of this study area. Therefore, the datasets used here could represent the spatial-

temporal features of soil moisture in the large gully. Fig. 2 shows the temporal dynamics of root zone soil moisture and its standard deviation. In general, surface soil moisture (0–20 cm) was highly dependent on rainfall events; it increased sharply following rainfall events and decreased slowly over the periods without rainfall (interstorm periods). In particular, surface soil moisture showed the lowest value over the interstorm periods while the highest value following relatively large precipitation. Unlike with surface layer, soil moisture in deeper depths showed lags in response to rainfall events, especially in the 60–80 cm. The lags might be a consequence of infiltration delay or insufficient rainfall amount into deeper depths. Moreover, S.D. also fluctuated during the study period.

### 4.2. Time stability analysis

#### 4.2.1. The feasibility of the new metric RMSE

Fig. 3 shows the time series of offset coefficients (MRD) for locations with different SDRD values. As shown there, locations having lower SDRD values showed less temporal variations of MRD, suggesting offset coefficients more approximate constants. According to Eqs. (5)–(7), these locations with lower SDRD values also would have lower RMSE values. Fig. 4 characterizes the relationship between the RMSE parameter used here and SDRD over the study period to examine the robustness of the RMSE for identifying time-stable locations. Generally, there were strong and positive correlations between them, with  $R^2$  ranging from 0.65 to 0.81 and an overall  $R^2$  of 0.72, and the strongest correlation was in the 20–40 cm ( $R^2$ , 0.81). These findings indicate that the RMSE parameter used in this study was useful for identifying time-stable locations. Despite the strong correlations between them, RMSE used here is a higher-efficiency metric than SDRD in estimating areal mean soil moisture since it not only can identify time-stable locations but also directly indicate the estimation error. It should be noted that the RMSE used here is different from the root mean square error of the relative difference of soil moisture introduced by Jacobs et al. (2004), which does not directly reflect estimation errors. It also differs from the MABE (mean absolute bias error) developed by Hu et al. (2010a), although the MABE can be used to identify time-stable locations and reflects estimation errors. Mathematically, the RMSE can be divided into two, variance (precision) and bias (accuracy), components, whereas MABE only reflects the bias error of estimated soil moisture, providing no indication of the precision of estimations. Therefore, we would recommend the RMSE here for time stability analysis when estimating spatial soil moisture averages.

#### 4.2.2. Identification and validation of time-stable locations across the root zone

RMSE values calculated for various depths at each of the sampling locations in the studied gully are shown in Fig. 5. Like with other studies (e.g., Brocca et al., 2010; Gao et al., 2011b; Hu et al., 2010a; Jacobs et al., 2004; Starks et al., 2006), the most temporally stable location would be identified and be used for spatial averages estimation. As in several other studies (Biswas and Si, 2011; Guber et al., 2008; Heathman et al., 2012; Hu et al., 2010b; Starks et al., 2006), we identified different time-stable locations at different depths. The location C4 was identified as the most stable location for the 0–20 and 40–60 cm with RMSE equaling to 0.68% and 0.72%, respectively. For the 20–40 and 60–80 cm, the most stable locations were B4 (RMSE, 0.58%) and A4 (RMSE, 0.72%), respectively. The results also showed that the time stability of soil moisture was depth-dependent. Across all sampling locations the mean RMSE (1.19%) was lowest for the 0–20 cm layer, higher for subsurface layers and highest (1.41%) for the 20–40 cm layer, implying that soil moisture time stability was lower in the

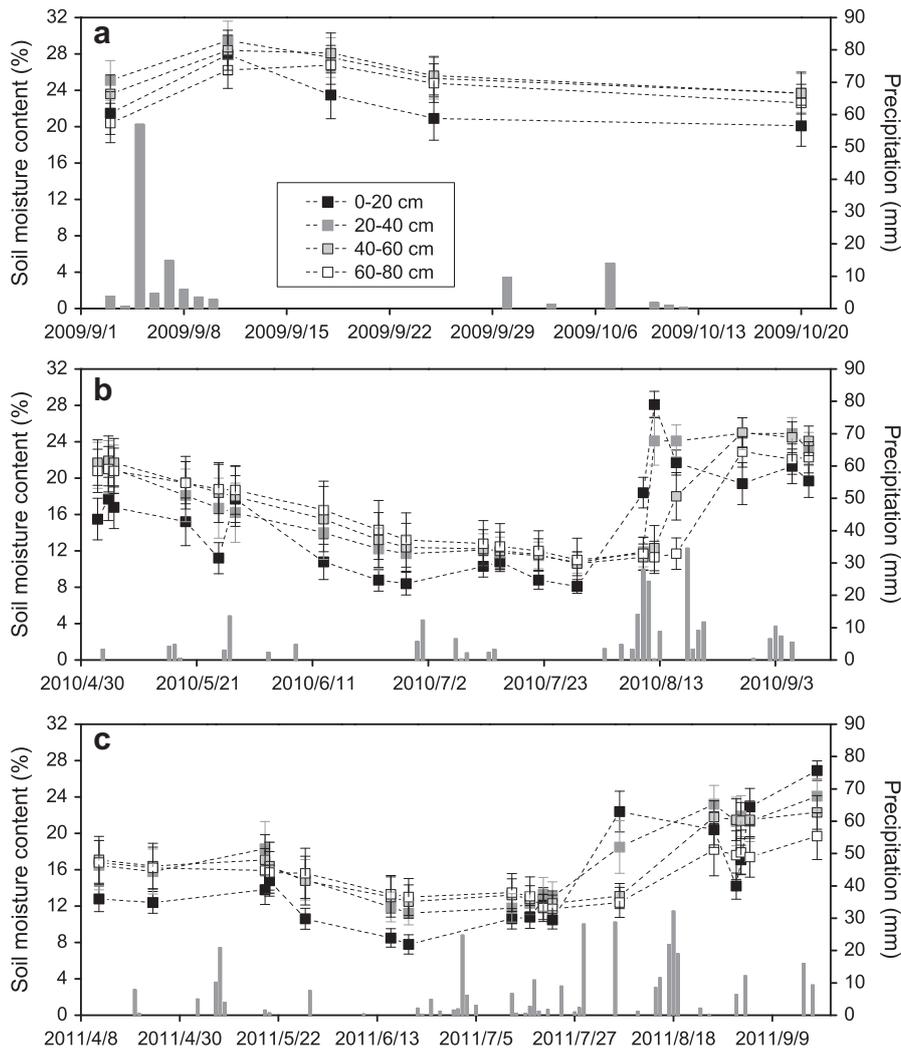


Fig. 2. Soil moisture time series at different depths over 3 years gullies: (a) 2009, (b) 2010, and (c) 2011. Error bars represent  $\pm$  one standard deviation.

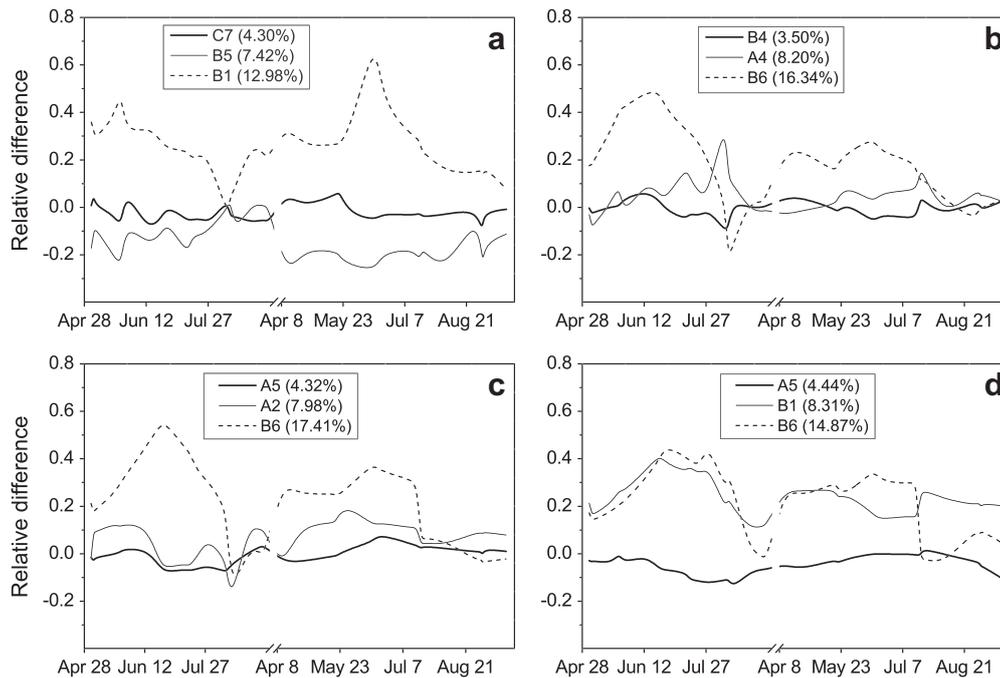
subsurface layers than in the surface layer. In contrast, Guber et al. (2008) found that time stability increased with depth. A possible explanation for the discrepancy is that the steep slopes and rough surface of gully sidewalls greatly affected the flow path of surface runoff and vegetation distribution, which resulted in diverse patterns of soil moisture infiltration for different sampling locations; hence the spatial structure of subsurface soil moisture in them may vary more with time. In addition, correlation analysis indicated that there were strong correlations among RMSE values for different layers (Table 4), suggesting that the time stability for a given layer is a good indicator of the time stability of other root zone layers.

The statistical results of soil moisture content at various depths from cross validation are shown in Table 2. Note that the 40 data subsets used to identify time-stable locations for each depth did not include data for the focal sampling day. Error statistics showed that errors (RMSE) rarely exceeded 1.5%. Moreover, approximately 50% of the error values (for 21 of the 41 days) were lower than 1.0% for the 0–20 cm layer. For greater depths, the numbers of days with <1.0% RMSE values were even higher (at least 28 of the 41 days). Further analysis showed that estimation errors were larger for the wet season than the dry season, and large errors were usually associated with days when rain fell. Jacobs et al. (2010) found similar patterns for validated time-stable locations in the Southern Great Plains hydrology experiments of 1997 (SGP97)

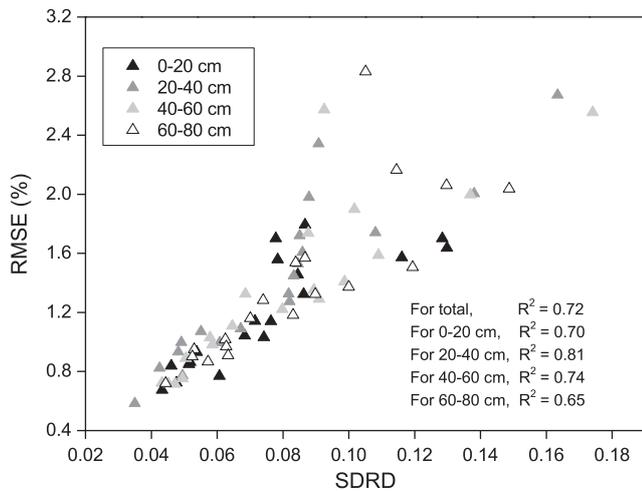
and 1999 (SGP99). This implies that additional soil moisture sampling is needed during wet seasons, especially for days when rain begins (Jacobs et al., 2010).

#### 4.2.3. Transferability of the MRD operator across root zone layers

A major goal of any time stability analysis of soil moisture is to find the most temporally stable location for estimating average spatial soil moisture contents. However, the most time-stable location usually varies for different layers (Guber et al., 2008; Heathman et al., 2012; Hu et al., 2010b; Pachepsky et al., 2005; Starks et al., 2006), despite significant correlations of RMSE values between different layers. In this part of the investigation, we tested if the MRD operator for a given layer could be reliably transferred across root zone layers. If so, only one MRD operator would be needed to estimate the mean soil moisture of different layers. Table 3 presents the statistical results of applying MRD operator for one layer to other root zone layers in order to test the vertical transferability of MRD operators. Although high  $R$  and near-zero MRD values were observed, root mean square error statistics were not satisfactory; specifically, when the MRD operator for one given layer was transferred to other layers, the root mean square error of spatial averages estimation increased by a factor of 1.5 to 2.5. This suggests that MRD operators cannot be transferred across to all of the other layers. However, Han et al. (2012) showed a successful transferability of MRD operator at two different layers (5 cm versus



**Fig. 3.** Temporal variations of relative difference for locations with different SDRD (standard deviation of relative difference) values (numbers in parenthesis) at various depths, (a) 0–20 cm, (b) 20–40 cm, (c) 40–60 cm, and (d) 60–80 cm.



**Fig. 4.** Relationship between root mean square error (RMSE) of estimated soil moisture and standard deviation of relative difference (SDRD) for soil moisture.

20 cm). This is probably because transferability was only conducted at two layers in their study. Our results suggest that when transferability is done among deeper depths, it is not necessarily transferable.

#### 4.2.4. Topographical features of time-stable locations

Correlation analysis was applied to examine the relationships between soil moisture time-stability (RMSE) and environmental variables, including soil properties of the 0–20 cm layer. Generally, the correlations, for various depths, were weak (Table 4), implying that use of one single environmental variable to identify time-stable locations would not be reliable. The most practically feasible way to identify time-stable locations, as yet, is to characterize sites in terms of environmental variables, such as soil texture, topographic attributes and land use/land cover properties (e.g., Gao

et al., 2011b; Grayson and Western, 1998; Hu et al., 2010b; Jacobs et al., 2004, 2010; Joshi et al., 2011; Teuling et al., 2006). However, the results may have serious limitations for application in areas other than those where the data were collected.

In this part of the study we attempted to classify time-stable locations based on topographic attributes, because locations with similar topography generally have relatively low variations in soil texture (silt loam) and other soil properties (Table 1). Moreover, we also excluded several commonly used topographic indices such as contributing area and topographic wetness index (Beven and Kirkby, 1979; Kim, 2012; Quinn et al., 1991) due to the relatively coarse-resolution DEM of the gully. With the purpose of a priori selection of time-stable locations via feasible and simple indicators, we mainly focused on topographic position (ridge, plane surface or pipe) here as it correlates more strongly with RMSE than other topographic attributes (Table 4), and is a particularly strong determinant of soil moisture in gully landscapes of the Loess Plateau (Gao et al., 2011a). The observed relationships between topographic position and RMSE are shown in Fig. 6. For the 0–20 and 20–40 cm layers, the lowest RMSE was observed in ridges and the highest in pipes, suggesting that time stability is highest at ridge locations. This might be explained as follows: soil moisture at dry period, according to Grayson et al. (1997), is mainly controlled by soil properties such as clay content. Because of the relatively low clay content for ridge locations (Table 1), soil moisture for ridge locations was lower than gully averages during dry days. Over wet period, soil moisture at shallow depth (0–20 and 20–40 cm in this study) is mainly dominated by topography. Due to the convex shape, steep slope (~75%) and low vegetation cover (Table 1), a large part of precipitation on ridges would lose in terms of runoff and thus soil moisture for ridge locations would be also lower than gully averages during wet days. Therefore, the relative difference values of soil moisture were stable for ridge locations independent of wetness conditions and thus showed relatively high time stability. This conclusion is consistent with the observations that four out of seven locations within 1% RMSE for the 0–20 cm layer (B9, A4, B8 and B7) were located in ridges (Fig. 5).

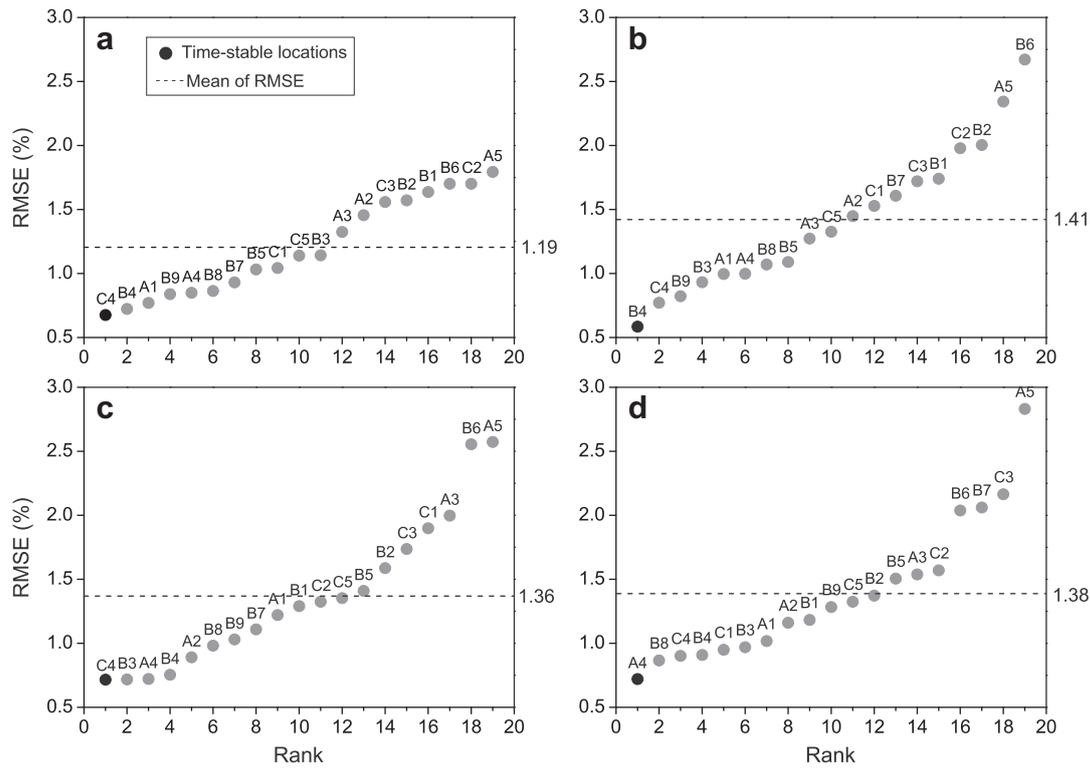


Fig. 5. Ranked RMSE for sampling locations in gullies based on time stability analysis, (a) 0–20 cm, (b) 20–40 cm, (c) 40–60 cm, and (d) 60–80 cm.

#### 4.3. Random combination analysis

When previous soil moisture data sets are not available, an alternative method is to use the random combination method introduced by Wang et al. (2008). We randomly selected from 1 to 19 upland locations for  $R = 1000$  replicates, and then compared the time series of the spatial mean soil moisture obtained from these locations with the benchmark time series. The determination coefficient ( $R^2$ ) and root mean square error for various depths when comparing benchmark time series and that obtained by averaging a different number of randomly selected locations were shown in Fig. 7. Generally, only several sampling locations were required to accurately estimate the spatial mean soil moisture in the gully. For the 0–20 cm layer, only three locations were needed to obtain a mean soil moisture time series with root mean square error  $< 1.0\%$  and  $R^2 > 0.98$ . For the other depths, only four locations were needed to reach this accuracy. Brocca et al. (2010) also found that data from just five locations (of 30 in total) were needed to obtain mean soil moisture temporal patterns with corresponding accuracy at the field scale. Furthermore, when eight locations and nine locations were randomly selected the root mean square error for the 0–20 cm and greater depths dropped to  $< 0.05\%$  and the  $R^2$  exceeded 0.99. These results imply that a moderate reduction in the number of sampling locations in this study site would not lead to a significant reduction of accuracy in mean soil moisture estimation. However, how many randomly selected locations are needed for an accurate estimation is site-specific and may vary from different study sites. It is also worthy noting that the error statistics of random combination analysis was based on 1000 replicates of random selection. In practice, it is impossible to conduct such a magnitude of replicates. Other information may be needed for the selection of sampling locations. Further study showed that the best choices of randomly selected locations for estimating soil moisture in large, ungauged gullies in the Loess Plateau using this approach would be in ridges, since ridge locations showed the highest time stability (Fig. 6).

The finding that only three or four locations are needed for accurate mean soil moisture estimation is probably due to the considerable time stability of soil moisture fields at our study site. In this sense, previous intensive soil moisture campaigns may be not necessary to obtain spatial–temporal soil moisture characteristics at this site. However, we should note that the accuracy of estimating areal means through the most time-stable locations was higher than that by averaging soil moisture values of three or four randomly selected location (Figs. 5 and 7). Furthermore, estimating areal mean soil moisture through only a few randomly selected locations can result in great uncertainty (high standard deviation) (Fig. 7) and thus repeats (maybe a large magnitude) are needed to diminish the uncertainty. Overall, time stability analysis is a robust method than random combination analysis for mean soil moisture estimation when previous soil moisture datasets are available whereas the latter may be a feasible alternative as previous soil moisture campaigns are not available.

#### 4.4. Estimation of spatial averages from precipitation observations

Initially, 11 time intervals during which there was no precipitation during the two study years were selected to calculate the temporal mean loss coefficient  $K$ . These 11 time intervals spanned wet and dry soil conditions. The results show that the coefficient ( $K$ ) changed with time, and values were relatively low during periods with dry soil conditions (Table 5). With a time step of 1 day, the empirical model introduced in Section 3.3 was then used to reproduce soil moisture time series for 2010 and 2011. The simulated and observed soil moisture time series for the 0–80 cm layers in these years are shown in Fig. 8a and b, respectively. Generally, the model successfully captured the temporal behavior of root zone soil moisture over the 2 years. The simulated time series were similar to the observed series, whereas several deviations were observed, especially during wet conditions. For instance, the model notably underestimated the soil moisture peak (August 28–September 9) in 2010, and overestimated the peak (August 27–Sep-

**Table 2**  
Statistical results of cross-validation analysis for the whole data sets.

Date	RMSE (%)			
	0–20 cm	20–40 cm	40–60 cm	60–80 cm
2009/9/3	1.64	1.30	1.20	1.35
2009/9/11	1.22	1.02	1.01	1.47
2009/9/18	0.84	1.05	0.92	1.15
2009/9/25	0.57	0.77	1.30	1.40
2009/10/20	1.62	0.60	0.73	0.91
2010/5/3	1.43	0.82	0.50	0.84
2010/5/5	1.28	0.77	0.85	0.80
2010/5/6	1.17	0.65	0.52	0.70
2010/5/19	1.17	0.70	0.65	0.50
2010/5/25	1.24	0.84	0.93	0.66
2010/5/28	1.41	0.72	0.74	0.65
2010/6/13	1.02	1.11	1.01	0.55
2010/6/23	0.68	0.75	0.65	0.99
2010/6/28	0.55	0.66	1.10	1.02
2010/7/12	0.60	0.71	0.94	0.76
2010/7/15	0.51	0.76	0.77	0.77
2010/7/22	0.64	0.46	0.69	0.67
2010/7/29	0.50	0.59	0.54	1.03
2010/8/10	1.22	1.08	1.30	1.18
2010/8/12	0.87	2.07	1.17	0.90
2010/8/16	1.11	1.34	0.89	0.90
2010/8/28	1.35	0.59	1.57	0.96
2010/9/6	0.85	0.69	0.26	0.60
2010/9/9	1.00	1.55	0.57	0.75
2011/4/12	1.19	0.85	0.58	0.54
2011/4/24	1.39	0.67	0.43	0.57
2011/5/19	0.80	0.65	0.64	0.36
2011/5/20	0.70	1.04	1.00	0.65
2011/5/28	0.62	1.14	1.21	0.77
2011/6/16	0.41	0.55	0.87	0.66
2011/6/20	0.22	0.67	0.37	0.46
2011/7/13	0.50	0.90	0.50	0.61
2011/7/17	0.87	0.45	0.45	0.70
2011/7/20	1.19	0.39	0.76	0.52
2011/7/22	0.70	0.76	0.90	0.62
2011/8/6	1.61	1.21	0.69	0.48
2011/8/27	0.94	1.04	0.55	1.41
2011/9/1	0.70	0.70	1.18	1.33
2011/9/2	0.65	0.45	0.64	1.55
2011/9/4	1.30	0.74	0.71	1.47
2011/9/19	1.01	1.04	0.78	1.26
Mean	0.96	0.85	0.81	0.87

tember 19) in 2011. The deviations between simulated and observed soil moisture storages in wet conditions could be attributed to three factors: (1) the constant loss coefficient assumption in the model being inconsistent with the real temporal variations of the coefficient; (2) deviations between the true precipitation and pre-

**Table 4**  
Results of correlation analysis between root mean square errors (RMSEs) of estimated mean soil moisture at various depths in gullies and their correlations with selected environmental variables. Significant correlations ( $P < 0.05$ ) are shown in italics and significant correlations ( $P < 0.01$ ) in bold.

Variables	RMSE20 <sup>a</sup>	RMSE40 <sup>b</sup>	RMSE60 <sup>c</sup>	RMSE80 <sup>d</sup>
RMSE20	1.00	<b>0.87</b>	<b>0.67</b>	<b>0.64</b>
RMSE40		1.00	<b>0.79</b>	<b>0.73</b>
RMSE60			1.00	<b>0.74</b>
RMSE80				1.00
Sand content	-0.43	-0.38	-0.24	0.01
Silt content	0.43	0.33	-0.04	0.13
Clay content	0.30	0.30	0.32	-0.08
Soil organic carbon	0.20	0.20	-0.02	-0.16
Soil total nitrogen	-0.21	-0.21	-0.47	-0.25
Slope	0.29	0.28	0.15	-0.23
Cos (aspect)	0.19	0.15	0.27	0.27
Relative elevation	-0.28	-0.09	0.00	0.22
Position	0.45	0.32	0.13	-0.06
Plant cover	0.06	0.11	0.12	0.22
Plant density	0.03	0.08	0.10	0.20

<sup>a</sup> RMSE20: RMSE for the 0–20 cm.  
<sup>b</sup> RMSE40: RMSE for the 20–40 cm.  
<sup>c</sup> RMSE60: RMSE for the 40–60 cm.  
<sup>d</sup> RMSE80: RMSE for the 60–80 cm.

cipitation records obtained from the nearby weather station, due to the steep slopes (average slope 84.5%) and rough surface (Fig. 1) at our study causing spatially uneven distribution of precipitation (Prudhomme and Reed, 1999); (3) the model failing to reflect the lateral flow path during wet season including that prolonged and high-intensity rainstorm in wet season could result in considerable amount of hillslope runoff, which may contribute to water storage in gullies, however, high-density rainstorm could also reduce the amount of water for infiltration through surface runoff over gully sidewalls. Overall, the model simulated the 2010 data significantly better than the 2011 data (determination coefficients: 0.968 and 0.880, respectively). Moreover, the prediction error for 2010 was 9.948 mm, significantly lower than the error (12.229 mm) for 2011. The poorer performance of the model in 2011 may have been because data for only four time intervals (of 11 in total) in 2011 were used to calculate the temporal mean loss coefficient. This implies that long-term soil moisture measurements at different hydrological years are needed to obtain the robust temporal mean loss coefficient  $K$ .

These results suggest that precipitation observations can provide estimations of mean soil moisture time series in gullies with certain accuracy. Using an analytical method, Pan et al. (2003) suc-

**Table 3**  
Statistics for vertical transferability of MRD operator across root zone layers.

Statistics		20–40 cm			40–60 cm			60–80 cm		
		R	RMSE	MBE	R	RMSE	MBE	R	RMSE	MBE
0–20 cm	Mean	0.987	0.014	0.005	0.988	0.013	0.005	0.983	0.013	0.002
	SD	0.012	0.004	0.008	0.006	0.005	0.007	0.014	0.004	0.007
20–40 cm	Mean	0.990	0.010	-0.001	0.990	0.010	0.002	0.990	0.014	0.000
	SD	0.004	0.002	0.007	0.005	0.002	0.004	0.003	0.005	0.012
40–60 cm	Mean	0.987	0.010	-0.002	0.988	0.010	-0.006	0.986	0.012	-0.002
	SD	0.011	0.003	0.005	0.010	0.003	0.009	0.009	0.003	0.010
60–80 cm	Mean	0.990	0.015	0.001	0.989	0.018	0.002	0.989	0.014	0.002
	SD	0.004	0.008	0.014	0.005	0.007	0.011	0.003	0.004	0.014

<sup>a</sup> SD: standard deviation.

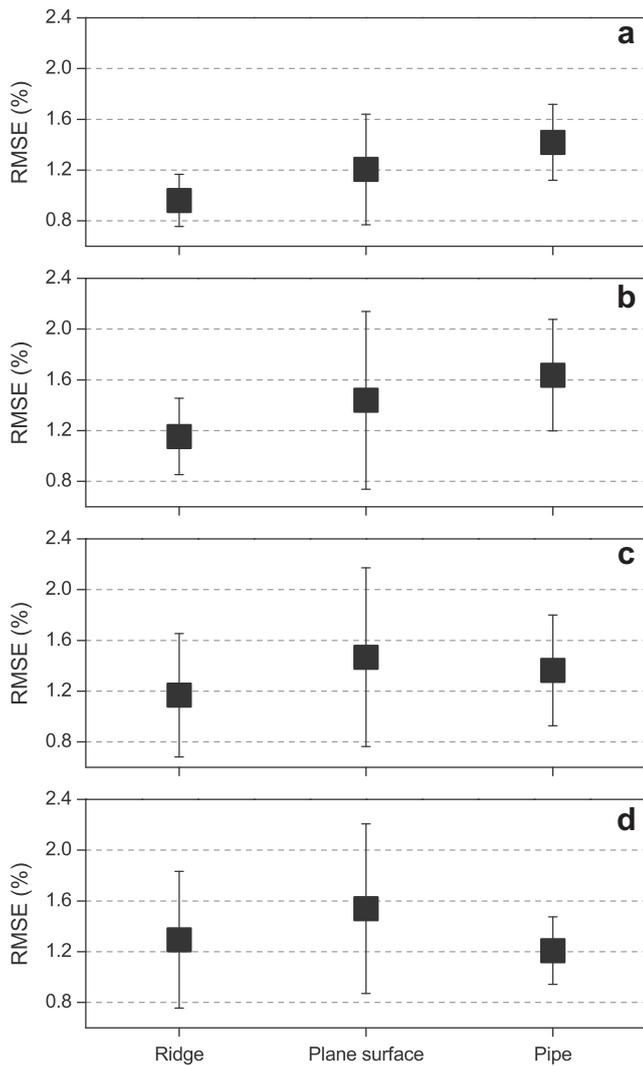


Fig. 6. RMSE at different depths for different topographic positions in gullies. Error bars represent  $\pm$  one standard deviation.

cessfully predicted surface soil moisture (0–5 cm) from precipitation observations. However, the applicability of the analytical method for estimating root zone soil moisture has not been evaluated. Although the empirical model presented here is simple and estimated root zone soil moisture with relatively good accuracy, a serious limitation is that initial soil moisture content is a required input parameter. Moreover, the relatively large estimation error in wet seasons suggest that only precipitation observations might be not enough to accurately predict root-zone soil moisture dynamics in large gullies, and time series of  $K$  should be modeled and included. In order to model temporal dynamics of  $K$ , future studies should collect more soil moisture observations during dry-down periods at both dry and wet seasons.

### 5. Summary and conclusions

In order to estimate spatial mean soil moisture in large gullies, statistical methods in terms of time stability analysis and random combination analysis were first used in the presented study to up-scale point measurements to spatial averages based on soil moisture datasets collected on 41 days over 3 years. We then developed an empirical model to test if precipitation is a good estimator of root-zone soil moisture time series in large gullies.

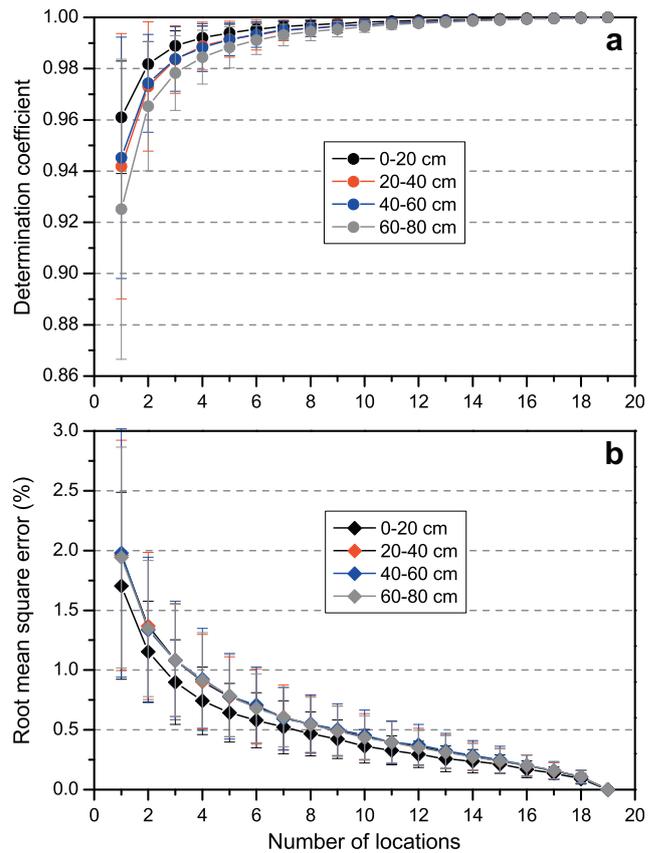


Fig. 7. Determination coefficient (a) and root mean square error (b) when comparing the benchmark time series with those obtained by averaging a different number of randomly selected locations at various depths. Error bars represent  $\pm$  one standard deviation.

Analysis of the correlation between SDRD and the new metric RMSE showed that the latter can robustly identify time-stable locations. Time stability analysis based on RMSE indicated that there is considerable soil moisture time stability in large gullies and spatial averages were accurately estimated through time-stable locations. Cross-validation analysis confirmed the temporal robustness of time-stable locations. However, MRD operators provided to have insufficient vertical transferability across root-zone layers, although significant correlations were found between RMSE values for different depths. This suggests that it is necessary to identify specific time-stable locations for given layers. In addition, differences in time stability were observed among various topographic positions, and greater time stability was observed in ridges than in pipes and plane surfaces.

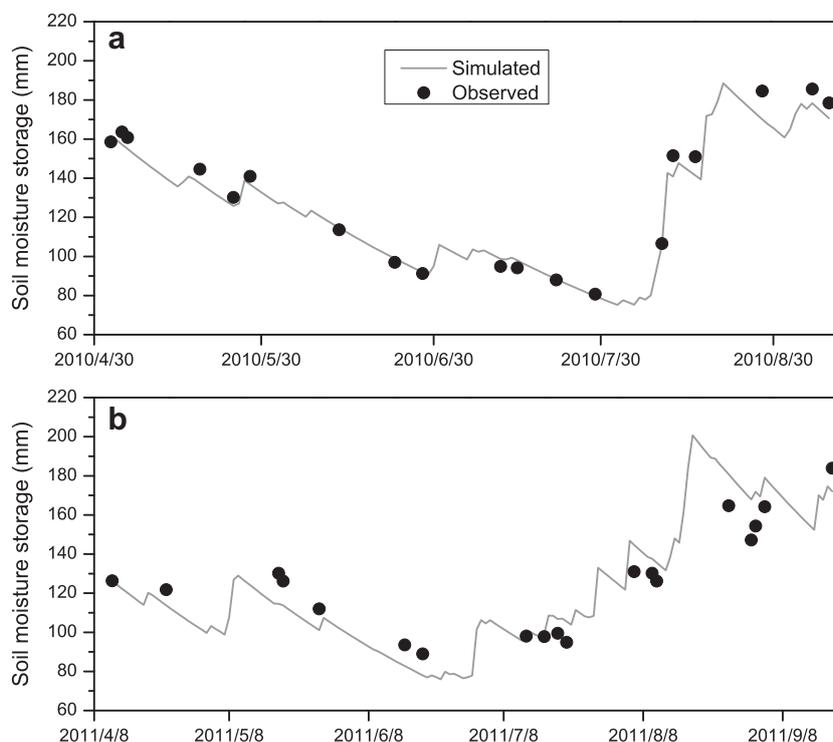
Random combination analysis revealed that not more than four randomly selected locations were needed to obtain the mean soil moisture in the gully within a good accuracy (root mean square error < 1.0% and  $R^2 > 0.98$ ). These results indicate that a moderate reduction in the number of sampling locations at our study site would not lead to a significant reduction in the accuracy of mean soil moisture estimates. Nevertheless, how many randomly selected locations are needed for an accurate estimation is site specific and may vary for different sites. However, we should note that time stability analysis could save more efforts compared to random combination analysis since only one single time-stable location can accurately estimate areal means. In addition, estimations through only several randomly selected would result in great uncertainty.

**Table 5**

Loss coefficient for different time intervals without rainfall and its temporal mean and standard deviation (SD).

Time interval	$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$	$T_8$	$T_9$	$T_{10}$	$T_{11}$	Mean	SD
$k$ ( $d^{-1}$ )	0.017	0.017	0.016	0.012	0.010	0.012	0.013	0.015	0.012	0.022	0.014	0.015	0.003

$T_1$ : May 5 2010–May 6 2010;  $T_2$ : May 19 2010–May 25 2010;  $T_3$ : June 13 2010–June 23 2010;  $T_4$ : June 23 2010–June 28 2010;  $T_5$ : July 15 2010–July 22 2010;  $T_6$ : July 22 2010–July 29 2010;  $T_7$ : September 6 2010–September 9 2010;  $T_8$ : April 12 2011–April 24 2011;  $T_9$ : May 20 2011–May 28 2011;  $T_{10}$ : June 16 2011–June 20 2011;  $T_{11}$ : August 27 2011–September 1 2011.

**Fig. 8.** Simulated and observed root zone soil moisture (mm) in the gully for 2 years, (a) 2010 and (b) 2011.

The simple empirical model used in this study successfully captured the root-zone soil moisture dynamics, suggesting that precipitation observations can provide relatively good estimation of mean soil moisture time series in large gullies. The constant loss coefficient assumption in this study may be mainly responsible for the significant deviations between simulated and observed values in wet seasons. Another serious shortcoming (the requirement for initial soil moisture as an input parameter) for this simple model would limit its applicability in areas where soil moisture is difficult to sample.

Based on the above, time stability analysis is a robust method for spatial average estimation in gullies if detailed previous soil moisture datasets are available and thus is recommended in this study area. Nonetheless, the simple empirical model would be recommended for prediction if we have only precipitation, initial soil moisture and soil moisture loss coefficient data. However, if no soil moisture data is available, application of random combination analysis may be a feasible alternative.

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