# The Quantum Mechanical Nature of the Law of Universal Gravitation and the Law of Coulomb's Interactions

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**Abstract** With the introduction of a complex number describing a matter, the law of universal gravitation and the *Coulomb*'s law of electrostatic interactions can be derived from the superposition of matter waves. More specifically, matter interactions can be described via the superposition of the associated *de Broglie* waves and their nature is determined by the quantum number $l$ of the superposing waves.

**Key Words** Universal Gravitation, Coulomb's Interaction, Quantum Mechanics, Matter Wave, Matter Representation

## Introduction

Currently, the theory of quantum mechanics is considered to be the foundation of modern science and technology. However, the nature of the universal gravitation and Coulomb's electrostatic interactions are not well understood or interpreted by the fundamental principles of quantum mechanics. Thus, the nature of both the universal gravitation and Coulomb's interaction remains empirical. We believe that the universal gravitation and Coulomb's interaction can only be understood on the basis of correct representation of matter structures. In this paper, we wish to report our initial work on this important topic.

## The Wave Packet Structure of a Matter

Our knowledge to the existence and behavior of a matter is obtained through the interactions with it. If we do not know how to interact with a matter, we simply do not know its existence. Therefore, matter interactions are of paramount importance in our knowledge of a matter. The question is: How does one matter interact with another? What governs the interaction? To answer these questions, we must first know the structure of the matter.

In quantum mechanics, the state of a particle moving in a force field defined by the potential function *V* is described by the *Schrödinger* equation:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi \qquad (1)$$

where $\hbar$ is the *Planck* constant *h* over $2\pi$, *E* the total energy, $\mu$ the reduced mass and $\psi$ the associated wave function of the system. If the mass of the force field is much larger than that of the particle, then $\mu$ can be substituted by the rest mass *M* of the particle. Obviously, the motion of the particle is described with respect to the center of the force field. So the kinetic and potential energies are the outcome of the interactions between the particle and the force field.

Now, consider a system that contains only one particle. In this case, the particle is free and we must use the particle itself as the frame of reference to describe its motion. Thus, the potential energy and the total energy must be zero. Hence the kinetic energy must also be zero. As a result, the *Schrödinger* equation becomes

$$-\frac{\hbar^2}{2M}\nabla^2\psi = 0 \qquad (2)$$

Rewrite this equation, we have

$$-\nabla^2(\frac{\hbar^2}{2M}\psi) = 0 \qquad (3)$$

Write

$$\psi' = \frac{\hbar^2}{2M}\psi \qquad (4)$$

then

$$\nabla^2\psi' = 0 \qquad (5)$$

which is the well-known *Laplace* equation. This simple derivation indicates that the description of the motion of a free particle is the *Laplace* equation. It is shown in many standard textbooks that equation (5) has a general non-zero solution in the spherical polar coordinates:

$$\psi' = \left(a_1 r^l + a_2 r^{-l-1}\right)P_l^{|m|}(\cos\theta)e^{im\phi} \qquad (6)$$

where $a_1$ and $a_2$ are constants that can be determined under proper boundary conditions; both $m$ and $l$ are quantum numbers, $|m| \le l$ and $l = 0, 1, 2, \cdots$; $P_l^{|m|}(\cos\theta)$ is the associated *Legendre* polynomials. On the basis of our knowledge, $\psi'|_{r\to\infty} \to 0$, then we have to set $a_1 = 0$ to satisfy this physical requirement. Thus,

$$\psi' = a_2 r^{-l-1} P_l^{|m|}(\cos\theta) e^{im\phi} \tag{7}$$

Combining (7) with (4) and writing $a_2 = f$, we obtain

$$\psi = f \frac{2M}{\hbar^2} r^{-l-1} P_l^{|m|}(\cos\theta) e^{im\phi} \tag{8}$$

It can be seen from equation (8) that $\boldsymbol{\psi}$ is proportional to the rest mass of the particle, which is in agreement with our knowledge. Thus, it can be understood that $\boldsymbol{\psi}$ is the wave function describing the matter waves associated with the particle of rest mass $M$. Obviously, the wave function has a quantum structure. The absolute magnitude, $|\psi|$, is the intensity of the wave and the contours of $|\psi|$ vs. $r, \theta$ and $\phi$ over all permitted $l$ and $m$ values in equation (8) describe the structure of the wave packet associated with the particle.

Since the solution of the *Laplace* equation is independent of the magnitude of the mass or the size of the particle, the change of the particle mass does not have any effect on the structural features of the associated matter waves, though the intensity of the associated matter waves will change with the magnitude of the mass. Hence the wave packet structure of any **free** object, from an electron to a planet, must have the same structural features described by the solution of the *Laplace* equation.

## The Superposition of Matter Waves: $\psi_1 * \psi_2$

According to *de Broglie*, matter can only be detected through the interaction with their associated waves. Thus, the interaction between two objects occurs when their associated waves superpose. Whether the interaction force is attractive or repulsive depends on the outcome of the wave superposition. If the waves superpose constructively, the two objects will attract each other; if the superposition is destructive, their interaction force will be repulsive.

Let $\psi_1$ and $\psi_2$ be the wave functions of two particles with rest mass $M_1$ and $M_2$, and charge $Q_1$ and $Q_2$, respectively. Thus, the intensity of the interaction between the two particles will be proportional to the absolute magnitude of both $\psi_1$ and $\psi_2$, or, their conjugate product: $\psi_1 * \psi_2$. According to equation (8), we have

$$\psi_1 * \psi_2 \big|_{l_1, l_2} = k \frac{M_1 M_2}{r^{(l_1 + l_2 + 2)}} P_{l_1}^{|m_1|} (\cos\ \theta) e^{-i m_1 \phi} P_{l_2}^{|m_2|} (\cos\ \theta) e^{i m_2 \phi} \quad (9)$$

To obtain the intensity of an interaction between two particles separated by a distance *r*, it is necessary to know both the *l* and the *m* values involved in the interaction. If $l_1 = l_2 = 0$, then $m_1 = m_2 = 0$. This means that only one type of interaction with the given quantum numbers is permitted. This permitted interaction is the long range interaction due to the fact that it is least sensitive to the change in distance between the two particles. The magnitude of this special type of interaction is

$$F\big|_{l=0} \propto \psi_1 * \psi_2 \big|_{l=0} = G \frac{M_1 M_2}{r^2} \quad (10)$$

where *G* is a proportional constant determined by the boundary conditions. Equation (10), which is the same as *Newton*'s Law of Universal Gravitation, is the consequence of the superposition of a special type of waves associated with the two interacting particles. This result demonstrates that <u>Universal Gravitation has the nature of wave superpositions</u>! Although $\psi_1 * \psi_2$ has been treated as the probability density of finding a particle in a designated area, it is, in fact, the intensity or the force of the interaction between two objects. Since this force is quantized, it provides the ground for the current understanding of the quantum probability interpretation.

## Matter Representation

Since *Coulomb*'s law of an electrostatic interaction has the same structural pattern of mathematics as *Newton*'s law of universal gravitation, we believe that *Coulomb*'s law shall have the same quantum mechanical origin, i.e., the nature of quantum wave superpositions. However, (8) does not contain a charge term. Therefore, the description of a matter with mass alone is incomplete and it is necessary to modify our general description of matter, i.e., matter shall be described with both mass and charge. Since the interaction between mass and charge cannot be observed experimentally, those two physical quantities may be combined through a complex number: ($M + iQ$), where $Q$ is the charge associated with the matter of rest mass $M$, which can be either positive or negative. Thus, after replacing $M$ with ($M + iQ$), (8) becomes

$$\psi = 2f\frac{M + iQ}{\hbar^2}r^{-l-1}P_l^{|m|}(\cos\theta)e^{im\phi} \tag{11}$$

As a consequence, the force of the long range interaction between two objects, i.e., the superposition of their corresponding matter waves with quantum numbers $l_1 = l_2 = 0$, and $m_1 = m_2 = 0$, can be represented as

$$\begin{aligned} F\big|_{l=0} \propto \psi_1 {}^*\psi_2\big|_{l=0} &= G\frac{(M_1 + iQ_1)(M_2 - iQ_2)}{r^2} = G\frac{M_1M_2 + iQ_1M_2 - iM_1Q_2 + Q_1Q_2}{r^2} \\ &= G\frac{M_1M_2}{r^2} + G\frac{Q_1Q_2}{r^2} + G\frac{iQ_1M_2 - iM_1Q_2}{r^2} \end{aligned} \tag{12}$$

The first term is the force of universal gravitation; the second term is the force of *Coulomb* interaction; while the third term is an imaginary number, which may be understood as the matter field of the entire newly formed two-particle system.

At this point, it is clear that both the universal gravitation and the *Coulomb* interaction are the consequence of the superposition of matter waves. In a microscopic interaction, since $G\frac{M_1M_2}{r^2} << G\frac{Q_1Q_2}{r^2}$, it is appropriate to neglect the force of universal interaction; while in a macroscopic system, we have $G\frac{M_1M_2}{r^2} >> G\frac{Q_1Q_2}{r^2}$. Thus, it is reasonable to ignore the *Coulomb* interaction. As we can see, the proposed complex representation of a matter will produce tremendous implications to our understanding of quantum mechanics. For example, it is now easy to understand why the interaction of the charges with the same sign is repulsive while the interaction between the charges of different sign is attractive.

## Conclusion

The introduction of a complex number ($M + iQ$) provides a complete description of a matter. We have discovered that the superposition of matter waves, i.e., the conjugate product of the two wave functions associated, respectively, with the two particles represents the force of an interaction between the two particles. With this complex representation, we have established the quantum mechanical foundation for both the law of universal gravitation and the law of *Coulomb* interaction. They are the outcome of the superposition of the long-range matter waves associated, respectively, with the interacting particles.