Classical Mechanics

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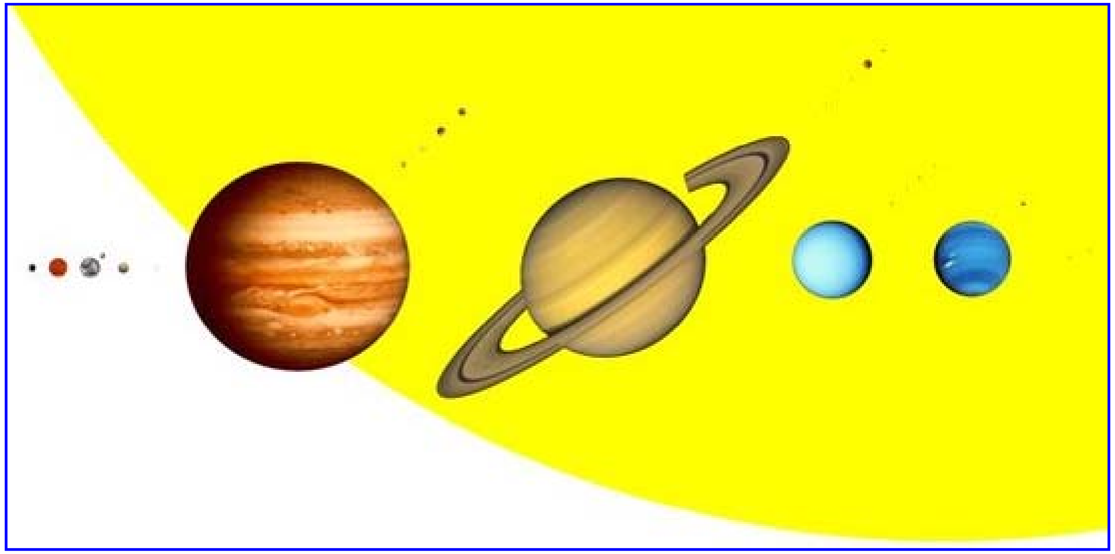
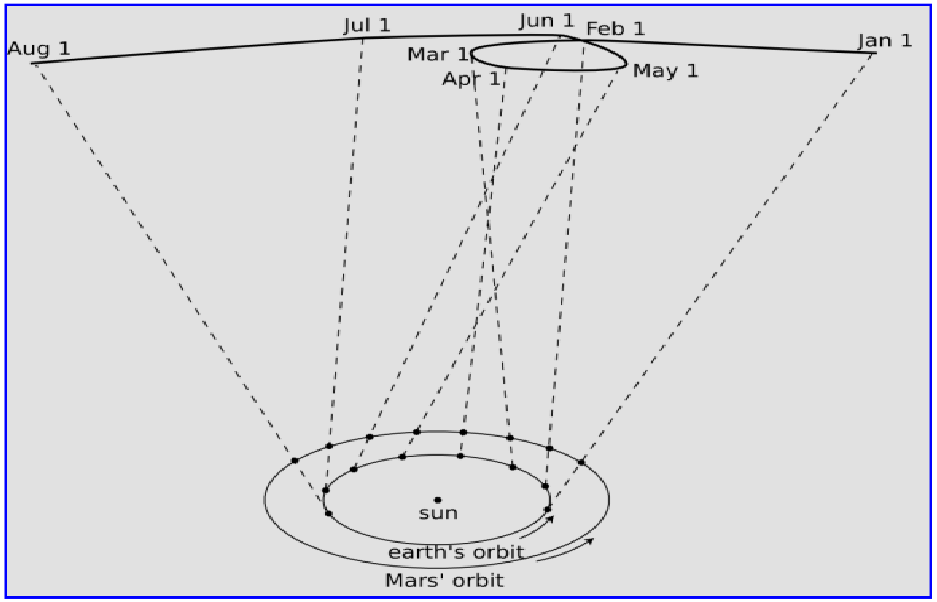
Early Astronomical Observations and Laws

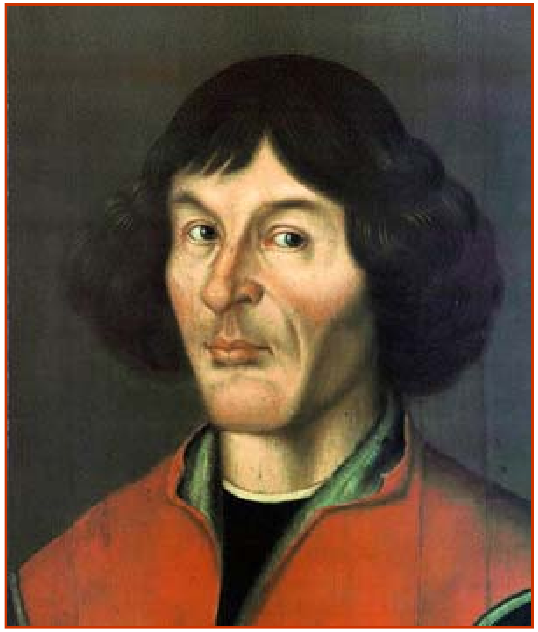


Modern illustrated solar system

Ptolemaic system



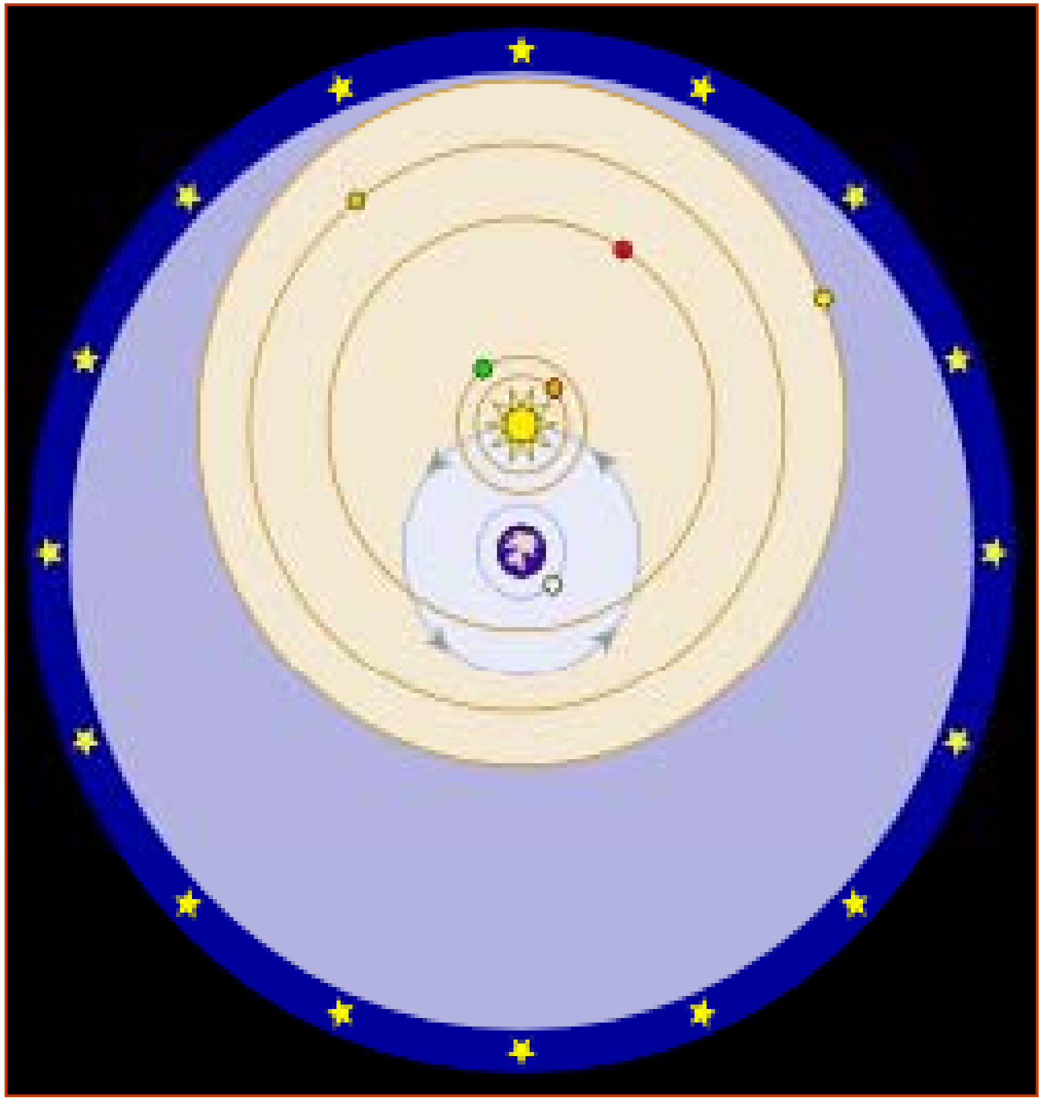
Kepler held to the heliocentric model of the solar system, and starting from that framework, he made twenty years of painstaking trial-and-error attempts at making some sense out of the data.

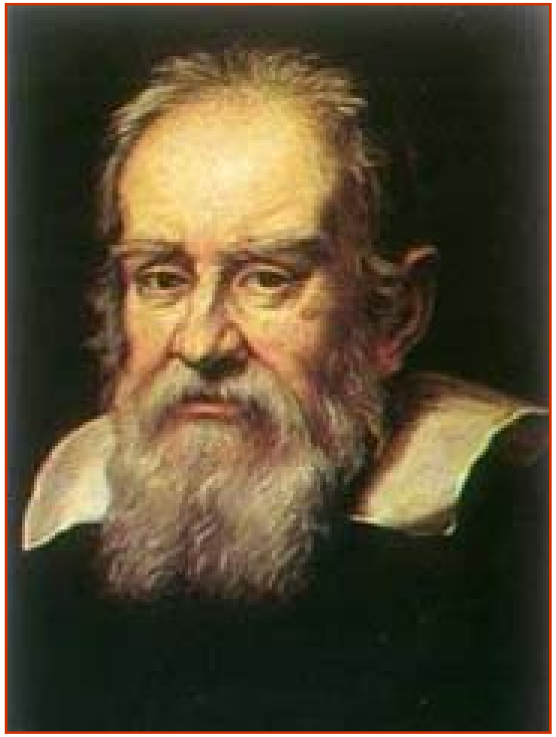


Nicolas Copernicus (1473 –1543), Polish mathematician and astronomer.

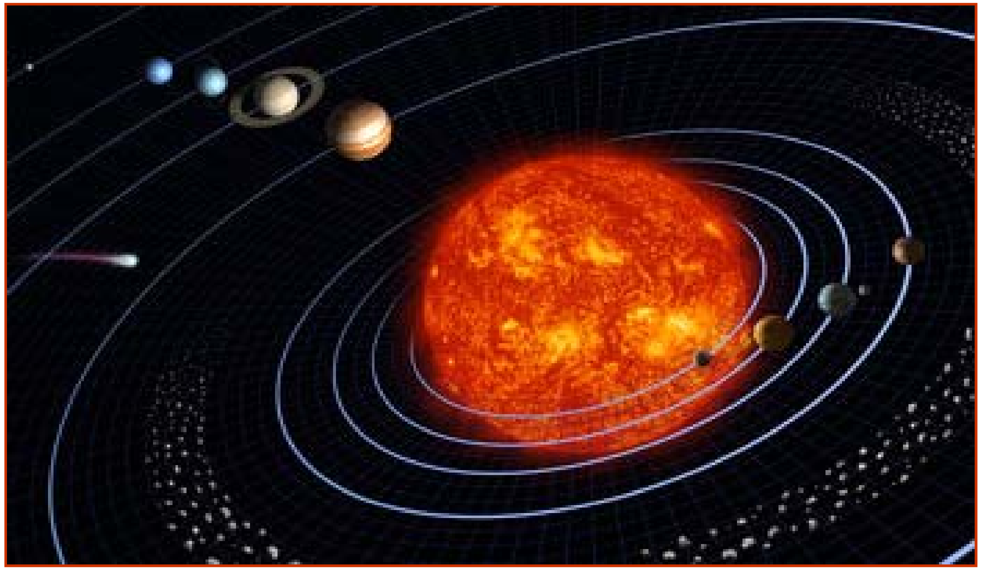
***<u>The sun is the center of our solar system</u>*!**

Tycho Brahe (1546 -- 1601), Danish astronomer.





Galileo Galileii (1564 –1642), Italian physicist, astronomer, and philosopher who is closely associated with the scientific revolution.



Kepler's first law (1609)

Kepler's elliptical orbit law: The planets orbit the sun in elliptical orbits with the sun at one focus.

Kepler's second law (1609)

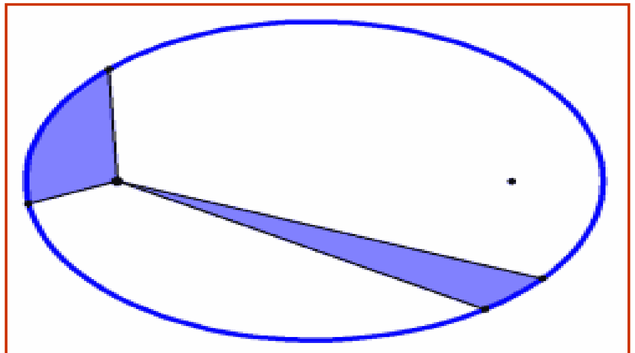
Kepler's equal-area law: The line connecting a planet to the sun sweeps out equal areas in equal amounts of time.

Kepler's third law (1619)

Johannes Kepler (1571 -- 1630), German mathematician and astronomer.

Kepler's law of periods: The time required for a planet to orbit the sun, called its period, is proportional to the long axis of the ellipse raised to the 3/2 power. The constant of proportionality is the same for all the planets.

Kepler's first law (1609)



Kepler's second law (1609)

$$T^2 \propto a^3$$

Kepler's third law (1619)

Result of ***Newton's law of gravitation*** and ***Newton's second law*** of motion

$$r = \frac{l^2 / GM}{1 + e\cos\theta}$$

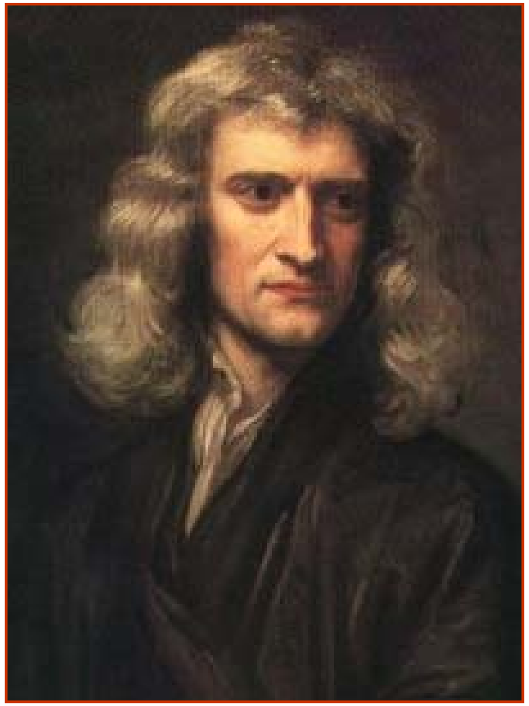
Result of ***Conservation law of Angular momentum*** (also ***Newton's second law*** of motion

$$dA = \frac{1}{2}\left|\vec{r} \times d\vec{r}\right| = \frac{\left|\vec{L}\right|}{2m} dt = \text{constant} \cdot dt$$

Result of ***Newton's law of gravitation*** and ***Conservation law of energy***

$$T^2 = \frac{4\pi^2}{G\left(m + M\right)} a^3$$

Newton's Fundamental Laws and Newtonian Mechanics



Law of Universal Gravitation (1660s)

Developing Calculus (1668)

Construction of a reflecting telescope (1668)

Newton's first Law (Inertia Law) (1687)

Sir Isaac Newton, PRS, (1643 –- 1727), English physicist, mathematician, astronomer, alchemist, inventor and natural philosopher who is generally regarded as one of the most influential scientists in history.

Newton's second Law of motion (1687)

Newton's third Law (1687)

Theory of light as corpuscle (1704)

Law of Universal Gravitation (1660s)

$$\vec{F} = -G\frac{m_1 m_2}{|\vec{x} - \vec{x}'|^3}(\vec{x} - \vec{x}')$$

$$G = 6.6726 \times 10^{-11}\mathrm{m}^3 \cdot \mathrm{s}^{-2} \cdot \mathrm{kg}^{-1}$$

$$g = 9.8067\mathrm{m} \cdot \mathrm{s}^{-2}$$

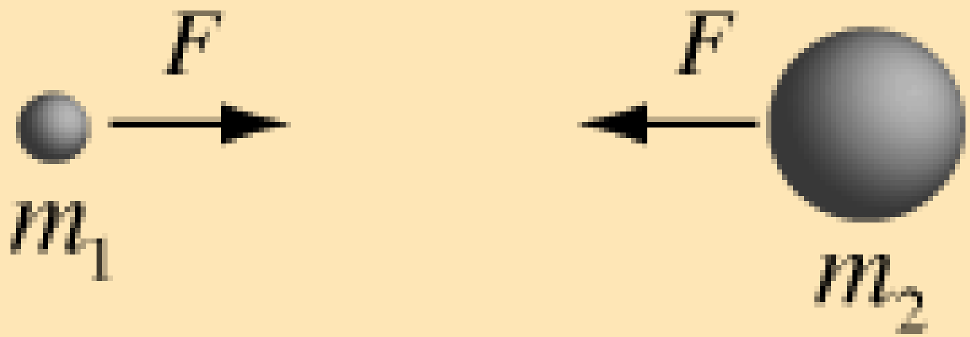
Newton's first Law (Inertia Law) (1687)

An object will remain *at rest* or in *uniform motion* in a straight line unless acted upon by an *external force*.

Newton's second Law of motion (1687)

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

Newton's third Law (1687)



Energy conservation law

$$\Delta E = 0$$

Momentum conservation law

$$\Delta \vec{P} = 0$$

Kinetic energy theorem

$$W = \Delta E_k$$

Momentum theorem

$$d\vec{P} = \vec{F}dt$$

Basic Mechanical Units

	SI Units (MKS)	(CGS)	U.S. Common
Length (L)	meter (m)	centimeter (cm)	foot (ft)
Time (T)	second (s)	second (s)	second (s)
Mass (M)	kilogram (kg)	gram (gm)	slug
Velocity (L/T)	m/s	cm/s	ft/s
Acceleration (L/T^2)	m/s^2	cm/s^2	ft/s^2
Force (ML/T^2)	kg m/s^2=Newton(N)	gm cm/s^2= dyne	slug ft/s^2=pound(lb)
Work (ML^2/T^2)	N m = joule (j)	dyne cm = erg	lb ft = ft lb
Energy (ML^2/T^2)	joule	erg	ft lb
Power (ML^2/T^3)	j/s = watt (W)	erg/s	ft lb/s

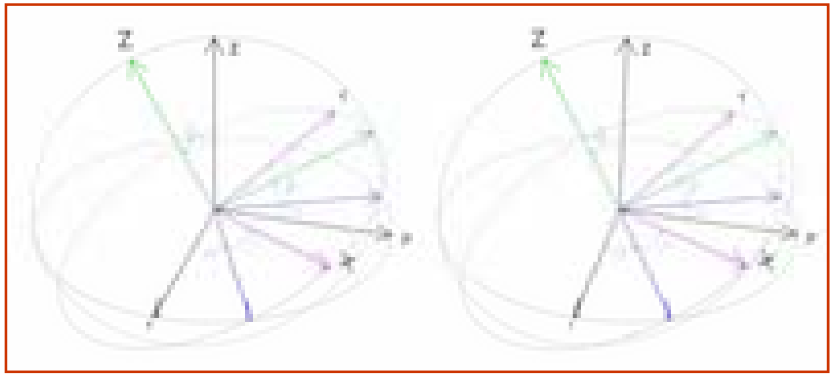
Unit Conversions

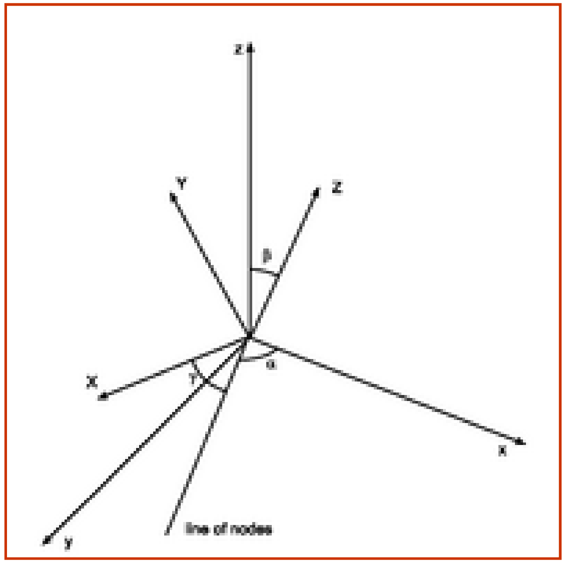
Euler's equation for rigid body

$$\left(\frac{d\mathbf{L}}{dt}\right)_{\text{relative}} + \omega \times \mathbf{L} = \frac{d\mathbf{L}}{dt} = \mathbf{N}$$

$$Mb_{G/O} \times \frac{d^2 R_O}{dt^2} + \frac{d\begin{pmatrix} \int y^2 + z^2\, dm & -\int xy\, dm & -\int xz\, dm \\ -\int xy\, dm & \int x^2 + z^2\, dm & -\int yz\, dm \\ -\int xz\, dm & -\int yz\, dm & \int x^2 + y^2\, dm \end{pmatrix}\begin{pmatrix}\omega_x \\ \omega_y \\ \omega_z\end{pmatrix}}{dt} = \sum_{j=1}^{N} \tau_{O,j}$$

Precession, nutation and spin.





Analytical Mechanics

$$\frac{\delta S}{\delta x_i(t)} = 0$$

principle of stationary action

Pierre de Fermat (1601 -- 1665), French mathematician.

Fermat's principle of least time

Pierre Louis Maupertuis (1698 -- 1759), Frence mathematician and philosopher.

Nature is thrifty in all its actions.

Gottfried Wilhelm Leibniz

(1646, -- 1716), German polymath, deemed a genius in his day and since.

Least action principle -- a "deep" principle of physics

Leonhard Euler (1707 -- 1783), Swiss mathematician.

Euler-Langrange equation

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

Euler formula

$$e^{i\theta} = \cos\theta + i\sin\theta\,.$$

Euler identity

$$e^{i\pi} + 1 = 0\,.$$

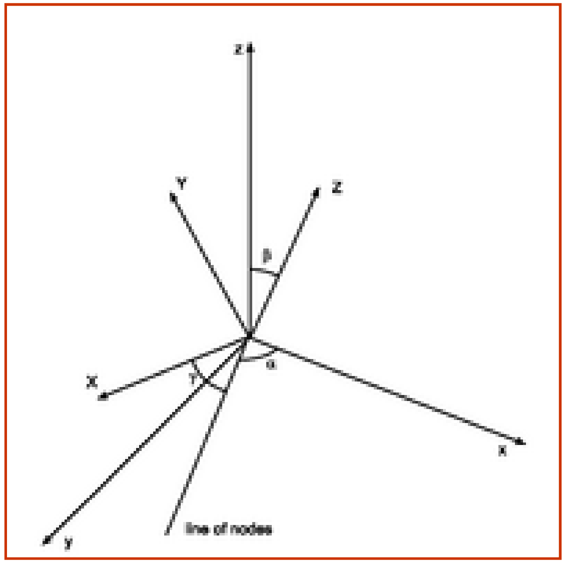
Basel problem – Riemann zeta function

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

Euler-Mascheroni constant

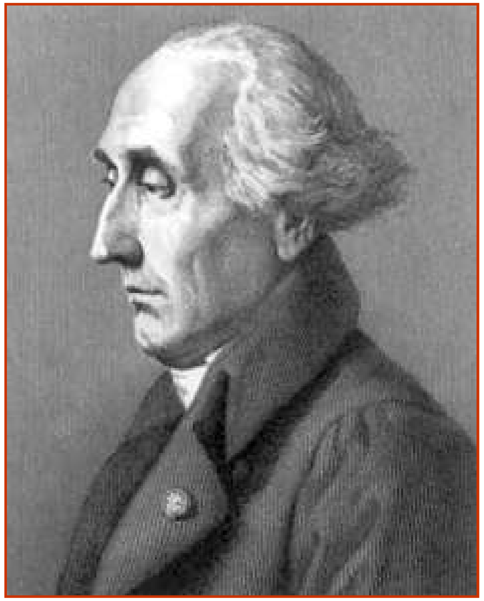
$$\gamma = \lim_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} - \ln(n)\right).$$

Euler angle



Euler number

$$e = \lim_{n\to\infty}(1 + 1/n)^n.$$



Joseph Louis Lagrange (1736 -- 1813), Italian-French mathematician, astronomer and physicist.

Lagrangian

$$L = L(q, \dot{q}; t)$$

Action

$$S = \int_{t_1}^{t_2} L(q, \dot{q}; t) dt$$

Euler-Langrange equation

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

Least action principle

$$\delta S = 0$$

William Rowan Hamilton (1805 -- 1865), Irish mathematician and astronomer.

Hamiltonian

$$p = \frac{\partial L}{\partial \dot{q}}$$

$$H(q,p;t) = \sum_i p_i \dot{q}_i - L(q,\dot{q};t)$$

Hamilton's equation

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Motion equation

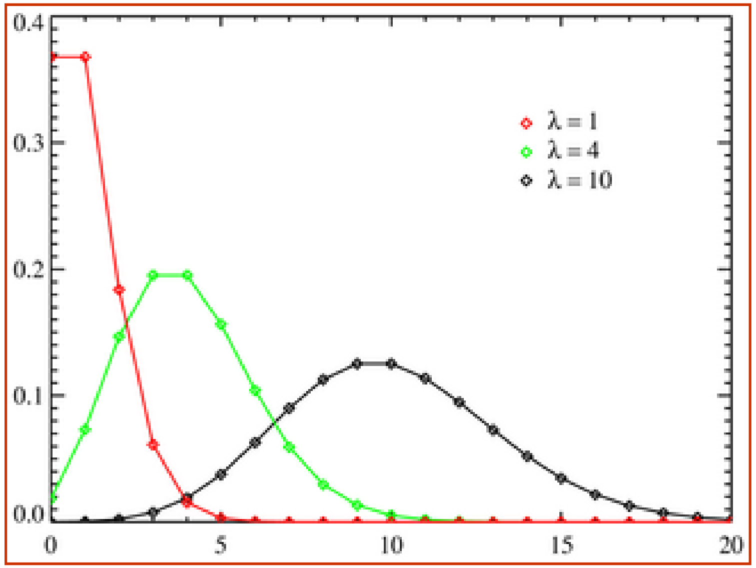
$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\}$$

Poisson bracket

$$\{f, H\} = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i} \right]$$

Hamilton-Jacobi's equation

$$\frac{\partial S}{\partial t} + H(q,p;t) = 0$$



$$f(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!},$$

Poisson distribution

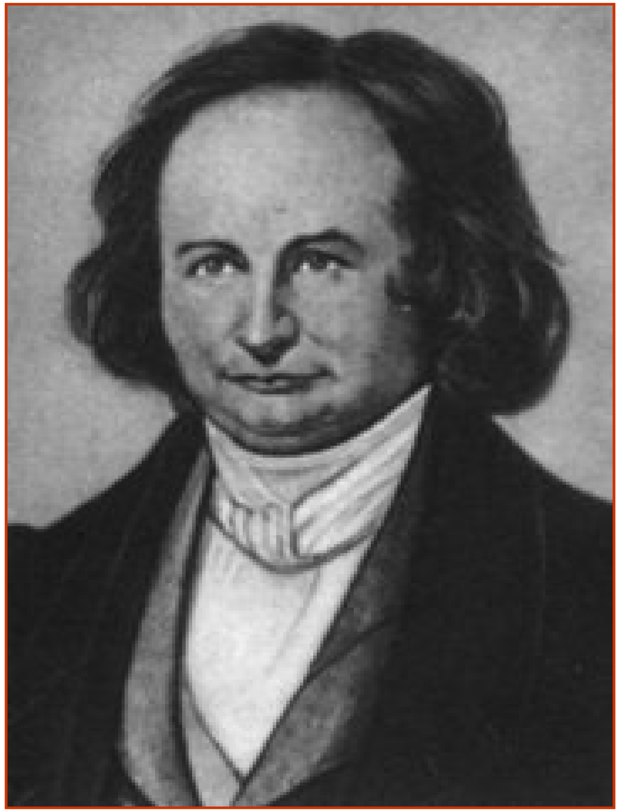
$$\nabla^2\varphi = f$$

Poisson's equation

Siméon Denis Poisson (1781 -- 1840), French mathematician, geometer and physicist.

$$\{f, H\} = \sum_{i=1}^{N}\left[\frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i}\right]$$

Poisson bracket



Carl Gustav Jakob Jacobi (1804 -- 1851), German mathematician.

Jacobian determinant

$$J_F(x_1, x_2, \cdots, x_n) = \frac{\partial(y_1, y_2, \cdots, y_m)}{\partial(x_1, x_2, \cdots, x_n)} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

Jacobi identity

$$[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0$$

Hamiltonon-Jacobi equation

$$\frac{\partial S}{\partial t} + H(q, p; t) = 0$$

Joseph Liouville (1809 -- 1882), French mathematician.

Liouville's theorem

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \sum_{i=1}^{d}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0.$$

$$\frac{\partial\rho}{\partial t} = -\{\,\rho, H\,\}$$

$$\frac{\partial\rho}{\partial t} + \hat{L}\rho = 0.$$

$$\hat{\mathbf{L}} = \sum_{i=1}^{d}\left[\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial}{\partial p_i}\right],$$

The distribution function is constant along any trajectory in phase space.

Density matrix equation

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[\rho, H]$$

END