Front. Phys. China, 2007, 2(3): 368－374
DOI 10.1007/s11467-007-0047-y

RESEARCH ARTICLE

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Solitary wave solutions of nonlinear financial markets: data-modeling-concept-practicing

Abstract This paper seeks to solve the difficult nonlinear problem in financial markets on the complex system theory and the nonlinear dynamics principle, with the data-model-concept-practice issue-oriented reconstruction of the phase space by the high frequency trade data. In theory, we have achieved the differentiable manifold geometry configuration, discovered the Yang-Mills functional in financial markets, obtained a meaningful conserved quantity through corresponding space-time non-Abel localization gauge symmetry transformation, and derived the financial solitons, which shows that there is a strict symmetry between manifold fiber bundle and gauge field in financial markets. In practical applications of financial markets, we have repeatedly carried out experimental tests in a fluctuant evolvement, directly simulating and validating the existence of solitons by researching the price fluctuations (society phenomena) using the same methods and criterion as in natural science and in actual trade to test the stock Guangzhou Proprietary and the futures Fuel Oil in China. The results demonstrate that the financial solitons discovered indicates that there is a kind of new substance and form of energy existing in financial trade markets, which likely indicates a new science paradigm in the economy and society domains beyond physics.

Keywords noneuclidean geometry, manifold, nonlinear, finance market, solitary wave, price fluctuation, data mining, gauge modeling, financial soliton

PACS numbers 89.65.Gh, 89.75.-k, 11.15.-q, 05.45.Yv

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Received December 27, 2006

1 Introduction

In the fundamental research of finance, people continuously utilize physics in their thinking and methods. Although established financial market models are mathematical models, there are also physical models that only use a different language when indicated [1]. In 1900, Louis Bachelier regarded the stock price rise and drop as a kind of random motion; the equation obtained is extremely similar to the equation describing a Brown grain motion [2]. After half a century, the theories of finance and economics (mathematics finance on pure mathematic) was founded. The mainstream financial investment model mainly has the Markowiz portfolio selection "not to put all eggs in one basket" (related coefficient ρ) [3]; Sharpe [4], Linther [5], Mossin capital asset prices model [6] (risk coefficient β); Samuelson and Fama efficient market theories on Kendall discovering the complete randomness of stock price fluctuation (Bachelier, Einstein diffusion coefficient D) [7]; Fischer Black and Scholes [8] and Merton the theory of options pricing and corporate liabilities (Black-Scholes model) [9], original property price (stock price) variety submitting distribute randomly (pulsating rate δ); and Ross the arbitrage theory of capital asset pricing about markets with no arbitrage assumption [10]. In the models mentioned above, there appears a common characteristic: a lot of nonlinear dynamics questions are hidden in the coefficient and assumptions in a financial market price randomly undulating process. Therefore, the basic structure of a financial market cannot be truly known and its trading pattern cannot be even determined completely, which leads to a total unawareness of the market price fluctuation.

The application of modern physics to research financial markets has already become a new method. Some authors applied statistical physics method to research the financial market's randomness, critical dynamics index sign analysis, chaos mechanism, fractal and bifurcation, and to determine if the financial market is a complete random fluctuation. The return rate does not obey the normal distribution or Gauss distribution, with the invariable characteristic of power-law and scale, which explains the financial market having the basic characteristic of self-organized criticality [11–20]. In recent years, some other authors have applied gauge theory [21–24], quantum mechanics [25, 26], field theory [27] etc. to research the pricing and to solve the different phenomena occurring in modern financial markets. These researches indicate that the application of modern physics methods to elaborate financial markets is possible: (1) If the financial market may make a physical system to look as if it is of a very great degree of freedom system. Now the evolution over time of economic systems can be observed and recorded continuously using modern computer trade systems. The massive records of the high frequency data and financial database are already available. There is a strict symmetry between the high frequency data provided by the market trade and physics research system on the space-time structure, which might respond to the evolution process and the behavior characteristic in the system. This system could achieve short or long term balance at a certain degree and is provided with some conserved quantities and conserved-law. These conserved quantities have already reflected certain symmetry. Therefore, the symmetry determines principle, which means nonlinear dynamics can be used to observe and analyze the problem of price fluctuation. (2) If an event space has a manifold structure, then the conclusion is a fiber bundle. From this, the fiber bundle complex geometry structure may completely describe the financial market complex system. In physics, physics which describes with the fiber bundle mathematics is a non-Abel gauge field, namely a Yang-Mills gauge field [28,29], whose representative is non-mediocre fiber bundle, whose bottom space structure must be a manifold, itself having a superposition local structure, which gives the new physical description [30]. Moreover, the nonlinear property of financial markets requires the need for a nonlinear description of non-Abel gauge field. (3) The mathematics tool in economics and finance has already surpassed the category of algebra quantity economics (traditional probability theory and stochastic differential equation), and has entered the category to explore the geometric form of market appearance structure (geometrization formulation). Also, all differential geometry mechanisms were introduced into quantum finance, which is a new domain.

The difficult problem of nonlinear financial markets is the domain that current financial academic theorists and financial engineers need to solve. The solution of this realistic question would be the basis for a solution of a difficult global problem. We put forward that the trade price fluctuation of financial markets (stock, futures) is nonlinear. The price described is only to classify it as a nonlinear equation system. However, the nonlinear scientific research indicates that the solitary wave is precisely the nonlinear equation solution. This article obtains the nonlinear specialty factors (financial solitons) using phase space reconstruction, data mining and numerical analysis for high frequency data on complex systems and using nonlinear dynamics (chaos, fractal, soliton), with the issue oriented (data- model-concept-practice) at the problems of price fluctuations in financial markets. We build a forward-looking and adapting evolutionary game model to open out the mechanism of the market's price fluctuations, carried out successfully by the repeatability experiments of China's financial markets (stock Guangzhou Proprietary and futures Fuel Oil) in a fluctuating evolvement, and validated the accuracy of the theory and the model.

2 Financial solitons (noneuclidean geometry) structure

2.1 Rationale of financial solitons (noneuclidean geometry) structure

The financial market time is a time of relativity, with retractability [31]. Uncertainty is implicated in fluctuation ratio and lucre, but information is expressed by intrinsic time. Financial market is described by the wave function of quantum theory [32]. The trade behavior relations of financial markets are a continuum, so differential games can be applied. After a trader's behavior in financial markets is transformed into the trade data form, the obvious correlation relevance is demonstrated, which accords with the continuum model. The trade price fluctuations trend in financial markets is tracked by martingale method and fixed point theory, and the keypoints are continuously retroactively recognized [33]. The price fluctuations are the nonlinear equation's system, without the analytic solution. It can only process the numerical analysis but may also have the traveling wave solution.

2.2 Financial markets on manifold

Based on the thought of continuum we discovered some significance symmetry invariants by carrying on many kinds of form processing of high frequency trade data in financial markets (stock and future), and reconstructing phase space, through certain coordinate and gauge transformation. This kind of conserved quantity has good stability like the soliton in physics, which we call the financial soliton [34]. The theory description about the space reconstructed differentiable manifold is as follows.

2.2.1 *Connection on main fiber bundle*

To define the connection on P from the bundle P on a complex vector bundle unitary group $U(n)$ on manifold M. The connection is educed its curvature form, which is indicated as Chern characteristic class c_k; this characteristic class is the generator of cohomology algebra $H^*(G_{m+n,m}(C),Z)$, namely cohomology class $c_k \in H^{2k}(G_{m+n,m}(C),Z)$. The mold P is the necessary and sufficient condition that there exist the topology invariance of differential coefficient manifold characteristic class.

Let (B,M,π,G) be an m dimension smooth manifold G-main bundle on M, $\dim G = r$. Lie group G is Lie transformation group of freely right acting on the space of bundle B. Moreover, B fiber position is held by the right acting of G on B. H is an m dimension smooth distributing on bundle space B, namely H is an m dimension smooth cutting subspace field on B. H is a connection on G-main bundle $\pi: B \to M$ under certain conditions.

A connection on $\pi: B \to M$ is a level distributing held the position by Lie group G right acting on B-main bundle.

2.2.2 *Discovering the Yang-Mills function in financial market*

Let (B,M,π,G) be the main fiber bundle on densification Riemannian manifold (M, g), $\tilde{\pi}: Ad(B) \to M$ is its concomitancy bundle, g is Lie algebra of structure group G. An $Ad(G)$ invariability inner product $\langle \cdot,\cdot \rangle g$ is given on g, but it has Riemannian structure $\langle \cdot,\cdot \rangle$ on vector bundle $\Lambda^r T^* M \otimes Ad(B)(r \geqslant 0)$, to cause arbitrary $\alpha, \beta \in \Lambda^r(T_p^* M) \otimes \tilde{\pi}^{-1}(\rho)$ of arbitrary $P \in M$,

$$\langle \alpha,\beta \rangle = \frac{1}{r!} \sum_{i_1,\cdots,i_r} \left\langle \Phi_b^{-1}(\alpha(e_{i_1},\cdots,e_{i_r})),\ \Phi_b^{-1}(\beta(e_{i_1},\cdots,e_{i_r})) \right\rangle g \tag{1}$$

where $\{e_i\}$ is the unit orthogonality frame field on M, $b \in \pi^{-1}(p)$. Due to $\langle \cdot,\cdot \rangle g$, $Ad(G)$ is invariable, the right side of Eq. (1) is independent of taking $b \in \pi^{-1}(p)$ as model. Therefore, for the arbitrary φ, $\psi \in \Gamma(\Lambda^r T^* M \otimes Ad(B))$, their (whole) inner product may be defined as follows:

$$(\varphi,\psi) = \int \langle \varphi,\psi \rangle \mathrm{d}V \tag{2}$$

simultaneously, if to order $\|\varphi\|^2 = (\varphi,\varphi)$, but it has $\|\varphi\|^2 \in C^\infty(M)$.

Now let $C(B)$ be the space constituted by all connections on main bundle $\pi: B \to M$, $H \in C(B)$, Ω is the curve connecting H, and Ω is the tensor form of $Ad(G)$ forming 2 orders on main bundle (B,M,π,G), thus it may be regarded as the smooth cross-section of vector bundle $\Lambda^2 T^* M \otimes Ad(B)$.

To define $J(H) = \frac{1}{2}\|\Omega\|^2$, a map (functional) has been confirmed $J: C(B) \to \Re$ by $H \to J(H)$. With $H \in C(B)$ given, if H is the critical point of Yang-Mills function J, then H is Yang-Mills connection on main bundle $\pi: B \to M$. The H curve form is a Yang-Mills field on Riemannian manifold (M, g), and as is now well known, it satisfies the Euler-Lagrange equation of the functional correspondence.

To set the existence of some Calabi-Yau space among special superstring vibration, and which is parallel among long-range interactions force field. The solitary wave solution of Yang-Mills function is solved with Martingale method, cohomology (K) theory, and backward stochastic differential equation mathematical techniques.

2.3 Modeling

We may obtain the comparative smooth fluctuation curve when the trend path of K-line with saw-dentate of financial market's trade price fluctuation is mapped (H_m). There is a valley turning point and peak turning point as well as an inflexion point on the fluctuation curve, namely the Picard fixed point existence. As the high frequency data of modern financial trade markets have guaranteed the function's continuity, we can use the financial soliton to process continuous-time (t) market price fluctuation (i.e., Brownian motion), and to approach stochastically the corresponding low or high point of the fluctuation (Fig. 1).

Using gauge transformation fundamental equation: non-Abel localization gauge transformation equation

$$\psi'(x) = \mathrm{e}^{\mathrm{i}q\theta(x)}\psi(x) \tag{3}$$

where $\psi(x)$ is nonlinear dynamics characteristic function, e is nature base of logarithms, i is imaginary unit, q is characteristic quantity of solitons, θ is parameter delineating the transformation.

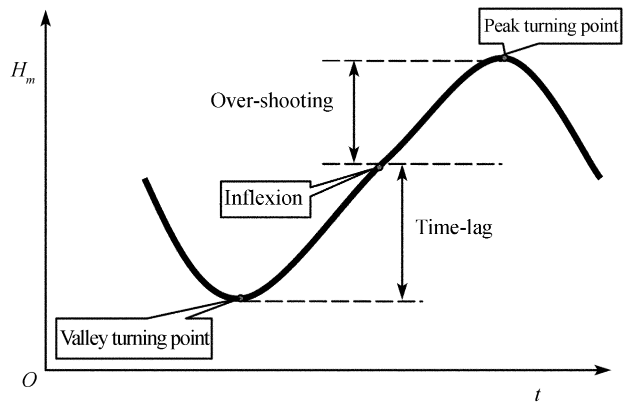

Fig. 1 The track map of trade price fluctuation in financial markets: fluctuation curve and keypoints recognized retroactively.

Using the Doob's emi-martingale decomposition theorem to track price which tends to change,

$$E\left[x_{n+1} \middle| y_0, \cdots, y_n\right] \geqslant x_n \quad (4)$$

when in an upward trend, $x_n^+ = \max\{0, x_n\}$; when in a downtrend, $x_n^- = \max\{0, -x_n\}$.

The self-organized criticality of price fluctuation decides the turning point (Picard fixed point) existence, thus, the fixed point theory is used,

$$d(f(x), f(y)) \leqslant \lambda d(x, y) \quad (5)$$

may predict approximation, identify and confirm the price fluctuation's peak and valley value.

It should be pointed out that the description of the above research is brought forward only as a theory and a research idea. For example, the relation among Eqs. (1), (2), (3) with Eqs. (4), (5) is not elaborated, and the functional relation and physical meaning of observable quantity in Eqs. (3), (4), and (5) is also not concretely elucidated. This is because some of the contents have already involved some sensitivity problem, such as (financial market investment strategy) intelligent property right, national finance strategic safety and so on. Of course, we also know that there still exist some imperfect instances that may need further thorough research among contents.

3 Data mining experimental research in financial markets

3.1 Design and step of experiment

3.1.1 *Experimental condition*

The stock Guangzhou Proprietary (600098): experimental data comes from the Shanghai stock exchange trade system to offer real-time stock markets, using Zhaoshang Securities Co., Ltd. Using the online stock trading system (VIP edition) to offer stock trading system data, the stocks are actually traded to test. The fundamental states are as follows: trade unit: 100 stocks/lot; trade cost: 4 ‰ of business volume; trade time: 9:30—11:30，13:00—15:00; in order to avoid distortion trade, aggregate bid for the right is not to join.

The futures Fuel Oil (0603→0604→0605): experimental data comes from the Shanghai Futures Exchange trade system to offer real-time futures markets, using Hunan Dayou Futures Broker Co., Ltd. Using online futures trading system to offer futures trading system data, the futures are actually traded to test. The fundamental states are as follows: trade unit: 5 ton/lot; trade cost: 9 yuan/lot; trade time: 9:00—11:30, 13:30—15:00; not to take part in aggregate bid for the right to avoid distortion trading.

3.1.2 *Data collection and treatment*

First, elementary numerical analysis is used to analyze history data of Guangzhou Proprietary or Fuel Oil, its fluctuation graph, and specific position of the fluctuation curve which they are on at the last minute is confirmed. During trading, the data of real-time trade markets, like prices, turnover, time interval, etc., are dealt by computer with ourselves-determination development the software. In the restructuring, the nonlinear special factors (financial solitons) are structured at the same time to couple with the curve constructed by the history data. The manifold special phase space is restructured and time series are treated [35].

3.1.3 *Methods*

Using the speculation principle on martingale and fixed point correlation methods; tracking financial solitons; the price fluctuation's key points (valley or peak fixed point and inflexion) are recognized, identified, affirmed, and reaffirmed by numerical analysis to the price fluctuation models; approaching the peak and valley of the price-waves in the stock and futures markets at random; and instructing purchase and sale trade. All the trade actions in this test are the decision-making results based on numerical analysis during modeling.

3.1.4 *Strategy*

Fund control is combined with dynamic programming of funds position management; the position is added with positive feedback, and elaborate firm offer is still included following the adaptation process: on price fluctuations of next class scale buying, buying to stop, or selling, selling to stop⋯.One finally becomes the minority winner in games by means of learning evolution with best optimizing the opportunity of the purchase and sale.

3.1.5 *Adaptability*

When a firm offer is operated, participant behavior must bring disturbance to all data systems of market games, and any perturbation is likely to develop resulting in the initial condition with chaos sensitivity. Therefore, the actual trading model is the numerical analysis after the disturbance is obtained, with interactions adaptability.

3.1.6 *Analysis and sum-up*

To close position, to state trade data, to analyze and to sum-up trade result, and give the numerical analysis and trade report.

3.2 Actual price trade test in Chinese financial markets

We actually traded to experiment for the stock Guangzhou

Fig. 2 The stock Guangzhou Proprietary (600098) actual price trade test.

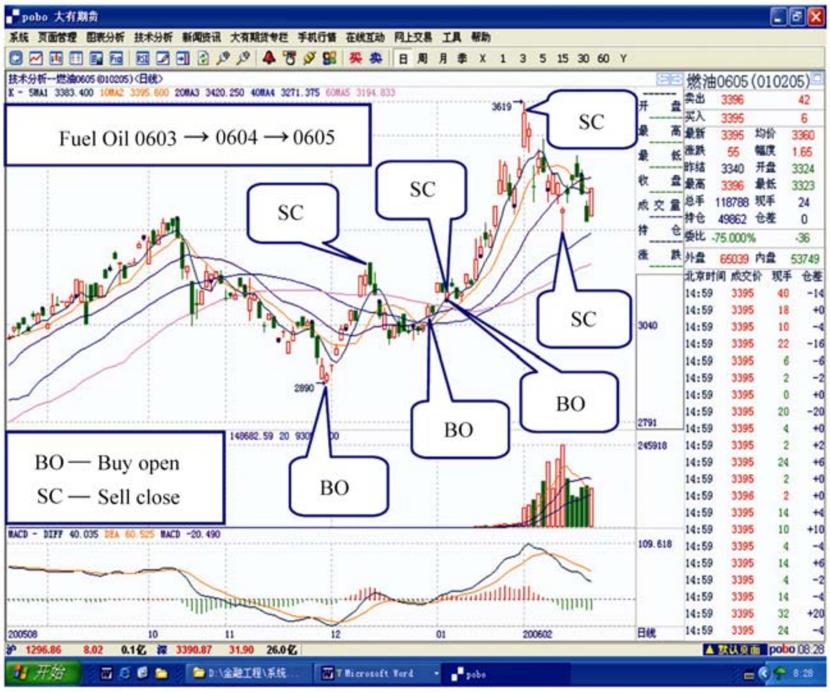

Fig. 3 The futures Fuel Oil 0603→0604→0605 actual price trade test.

Proprietary (600098) and the futures Fuel Oil 0603→0604→0605 in China. The trend of price fluctuations can be availably tracked by martingale method and fixed point theorem, and the valley or peak fixed point of fluctuations can be reckoned to approach, identified and affirmed, thereby realizes "buy low and sell high" (Table 1, Fig. 2, Table 2, Table 3, Fig. 3).

Table 1 The data of stock Guangzhou Proprietary (600098) actual price trade test.

Date	Side	Stocks	Price
20070119	Sell	6 100	6.740
20070119	Sell	17 900	6.740
20060614	Buy	12 600	4.060
20060518	Buy	11 400	4.410

Table 2 The trade result and its state of stock Guangzhou Proprietary actual price trade test (unit: Yuan).

	Trade	Maximum price	Minimum price	Average price	Total trade money
Trade result	Buy	4.410	4.060	4.235	101 640
	Sell	6.740	6.740	6.740	161 760
Stat. of result	Range: 80.77 % Turnover rate: 219.62 % Total turnover: 24 000 stocks Income: 161 760－101 640＝60 120 Yuan Yield: 60 120/101 640＝59.1 %; Operation period: 8 months				

Table 3 The data of futures Fuel Oil 0603→0604→0605 actual price trade test.

[A]fu	Date	Side	*	Lots	Price	Profit
0603	20051130	Sell	O	15	2 911.00	0.00
0603	20051202	Buy	O	45	3 070.76	0.00
0603	20051202	Buy	C	15	2 980.00	－10 350.00
0603	20051212	Sell	C	35	3 107.14	23 400.00
0603	20051229	Buy	O	5	3 055.14	0.00
0603	20060106	Sell	O	15	3 103.00	9 400.00
0604	20060106	Buy	O	15	3 107.00	0.00
0604	20060206	Sell	C	15	3 585.47	71 770.00
0605	20060206	Sell	C	10	3 589.00	400.00
0605	20060206	Buy	O	35	3 562.57	0.00
0605	20060207	Sell	O	5	3 504.00	0.00
0605	20060207	Buy	C	5	3 530.00	－1 300.00
0605	20060207	Sell	C	5	3 499.00	－4 300.00
0605	20060216	Sell	C	20	3 318.25	－45 500.00

*O－Open, C－Close.

3.2.1 *Stock Guangzhou Proprietary (600098)*

The circulation amounts to 6.80 billion Yuan. It specializes in electric power, energy, materials circulation etc.

3.2.2 *Futures Fuel Oil 0603→0604→0605*

The fluctuation operation was achieved by moving the warrants in actual price trade testing the futures. Total income is 43 520 Yuan, and operation period is 79 days.

3.3 Experiment result analysis

The financial solitons can follow the price fluctuation of stock and futures effectively and process the trade test of repeated experiments in fluctuant evolvement because it considers the following factors well: (1) Symmetry breaking, criticality and quantum condensed of the system researched. (2) The energy self-trapping of the system, namely the emergence of the soliton is always accompanied with lower system energy, and the energy transforms the soliton to binding energy. (3) The soliton form depends entirely on ① system state of motion, ② dispersion effect, ③ nonlinear action, ④ interactive dissipation, i.e., the financial solitons have respectively opposite independent character.

For more details on the model for researching China's financial markets (stocks and futures) and the modeling of trade price fluctuations in financial markets and so on, please see Changsha Workroom of Nonlinear Special Dynamics: http://solitonfund.blogchinese.com.

4 Conclusion

In this paper we discovered the Yang-Mills function of financial markets, obtained significant conserved quantity and derived financial solitons by transforming correspondent space-time and non-Abel localization gauge symmetry. It shows that there is a strict symmetry between manifold fiber bundle and gauge field in finance markets, that the differentiable manifold theory had preferably described the relation of internality interactions in finance markets (securities, futures), and that the significance conserved quantity can be educed by manifold symmetry transformations. The characteristic of availability tracking markets price fluctuations explains that the conserved quantity is the traveling wave solution (soliton) of stock and futures price fluctuations. The financial solitons discovered indicate that there is a kind of new substance and form of energy existing in financial trade markets (securities, futures), which is the emergence of interactions under complex systems. The more profound significance is that, since Einstein's unified field theory, the unified theory of interactions has been greatly extended, which is likely to indicate a new science paradigm in the economy and society domains beyond physics.

We successfully carried out actual price trade test to Chinese stocks and futures by applying the financial solitons (noneuclidean geometry) structure, which explains that "gauge modeling" is feasible, correct or successful. The financial theory no longer is an armchair strategist and is separated from the actual. Under the system of a market economy where the worth of information (speculation) submerges in the high frequency data of financial markets, the knowledge discovery of database comes true and is provided with information black box visualization in information asymmetry by effective data mining. The market and price mechanism is the only possible way that lets every one acquire other's knowledge, and gain own wealth by using other's knowledge.

Acknowledgements This work was supported by the Soliton Foundation.

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