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Signal frequency and parameter estimation for power systems using the hierarchical identification principle

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ABSTRACT

Estimating the fundamental frequency and harmonic parameters is basic for signal modelling in a power supply system. This paper presents a gradient based algorithm and a least squares based algorithm to estimate the fundamental frequency, the amplitudes and the phases of harmonic waves according to the voltage/current samples of a power system. Differing from the existing parameter estimation algorithms either for power quality monitoring or for harmonic compensation, the proposed algorithms are based on the hierarchical identification principle and are able to estimate the fundamental frequency, the amplitudes and the phases of harmonic waves simultaneously. In addition, the proposed algorithms are in the recursive form, which is suitable for on-line implementation. The simulation results verify the effectiveness of the proposed algorithms.

1. Introduction

Signal modelling and parameter estimation have received much research attention in signal processing and identification [1,2]; for example, Ding et al. presented time series AR modelling with missing observations based on the polynomial transformation [3]; Han et al. proposed an auxiliary model identification method for multirate multi-input systems based on least squares [4] and a multi-innovation stochastic gradient algorithm for multi-input multi-output systems [5]; Liu et al. developed a multi-innovation stochastic gradient algorithm for multiple-input single-output systems using an auxiliary model [6]; they also give an auxiliary model based recursive least squares algorithm for parameter estimation in non-uniformly sampled systems [7,8].

Recently, Wang and Ding presented an extended stochastic gradient identification algorithm for nonlinear systems [9–11]; Ding et al. proposed a multi-innovation stochastic gradient algorithm for linear regression models [12] and derived an auxiliary model based extended stochastic gradient algorithm for output error moving average models [13]. In this paper, the frequency and amplitude estimation problems will be studied for periodic signal modelling.

The frequency of an electrical power system is a significant operating parameter; not only does it indicate the dynamic balance between power generation and power consumption, but also the frequency deviation results in a component-reactance change which influences different relay functionality [14]. Thereby the system frequency can be regarded as a reasonable indicator to detect abnormal operating conditions. Several algorithms have been developed, such as the zero-crossing based methods [15,16], the discrete Fourier transform and its modifications [15,17], the Kalman filtering method [18], and the phase-locked loop [19], aiming to estimate power system frequencies. In addition, the proliferation of power electronic devices induces harmonic pollution to power systems, causing operational problems such as signal interference and malfunction of relays [20]; therefore harmonic measurement and compensation have become one of the most significant aspects of power quality monitoring and control [21]. In the time domain, the method based on the

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instantaneous reactive power theory has been widely adopted for harmonic compensation, in which the harmonic signals are extracted from both voltage and current measurements [22]. In the frequency domain, besides the analytical Fourier methods such as the discrete Fourier transform [23], Kalman filtering [24] allows optimal real-time estimation of harmonic parameters. These frequency domain methods use either the voltage or the current signal and can detect specific harmonic components [25].

On the basis of the work in [26,27], this paper presents a gradient based algorithm and a least squares based algorithm to estimate the fundamental frequency and the harmonic parameters of a power system according to the samples of the voltage/current signal. The proposed algorithms are able to estimate the fundamental frequency, the amplitudes and the phases of the harmonic waves simultaneously using the hierarchical identification principle and are easy to implement on-line.

The rest of this paper is organized as follows. Section 2 gives the parameter and frequency identification model for periodic signals. Section 3 derives a gradient based estimation algorithm for the fundamental frequency, and amplitudes of the fundamental wave and harmonics. Section 4 presents a least squares based frequency and parameter estimation algorithm using the hierarchical identification principle. Section 5 gives an illustrative example to show the effectiveness of the least squares based algorithm. The conclusions of the paper are summarized in Section 6.

2. Identification models for periodic signals

A distorted electric signal $s(t)$ from an AC power system can be expressed in the form of Fourier series:

$$s(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t), \tag{1}$$

where ω is the fundamental frequency of the AC system; n denotes the harmonic index, and a_n, b_n $(n = 1, 2, 3, \dots)$ are the Fourier coefficients of the nth harmonic; a_0 denotes the system DC component; here we assume that $a_0 = 0$, for there is generally no DC component for a typical AC power system. In addition, finite harmonics are generally substituted for the infinite harmonics in practice. Thus with the assumption that the largest harmonic index is N, Eq. (1) can be rewritten as

$$s(t) \simeq \sum_{n=1}^{N}(a_n \cos n\omega t + b_n \sin n\omega t), \tag{2}$$

or

$$s(t) = \sum_{n=1}^{N}(a_n \cos n\omega t + b_n \sin n\omega t) + v(t), \tag{3}$$

where $v(t)$ is the approximate error and may be regarded as a stochastic white noise with zero mean.

For simplification, let $s_k := s(t_k)$, $v_k := v(t_k)$; then, from Eq. (3), we have

$$s_k = \sum_{n=1}^{N}(a_n \cos n\omega t_k + b_n \sin n\omega t_k) + v_k. \tag{4}$$

Define the parameter vector $\boldsymbol{\theta}$ and information vector $\boldsymbol{\varphi}(\omega, k)$ as

$$\boldsymbol{\theta} := [a_1, b_1, a_2, b_2, \dots, a_N, b_N]^{\mathrm{T}} \in \mathbb{R}^{2N},$$
$$\boldsymbol{\varphi}(\omega, k) := [\cos \omega t_k, \sin \omega t_k, \cos 2\omega t_k, \sin 2\omega t_k, \dots, \cos N\omega t_k, \sin N\omega t_k]^{\mathrm{T}} \in \mathbb{R}^{2N}.$$

Then Eq. (4) can be rewritten as

$$s_k = \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\boldsymbol{\theta} + v_k. \tag{5}$$

The objective of this paper is to present new identification algorithms to estimate the frequency ω and unknown parameter vector $\boldsymbol{\theta}$ according to the samples $s(t_k)$ $(k = 0, 1, 2, \dots)$ of $s(t)$.

3. The gradient based estimation algorithm

Define a quadratic criterion function:

$$J_1(\boldsymbol{\theta}, \omega) := \left[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\boldsymbol{\theta}\right]^2. \tag{6}$$

The partial derivatives of $J_1(\boldsymbol{\theta}, \omega)$ with respect to $\boldsymbol{\theta}$ and ω are

$$\begin{cases} \dfrac{\partial J_1(\boldsymbol{\theta}, \omega)}{\partial \boldsymbol{\theta}} = -2\boldsymbol{\varphi}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\boldsymbol{\theta}], \\ \dfrac{\partial J_1(\boldsymbol{\theta}, \omega)}{\partial \omega} = -2\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}'_{\omega}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\boldsymbol{\theta}], \end{cases}$$

where

$$\boldsymbol{\varphi}'_{\omega}(\omega,k) = \frac{\partial \boldsymbol{\varphi}(\omega,k)}{\partial \omega} = [-t_k \sin \omega t_k, t_k \cos \omega t_k, -2t_k \sin 2\omega t_k, 2t_k \cos 2\omega t_k, \dots, -Nt_k \sin N\omega t_k, Nt_k \cos N\omega t_k]^{\mathrm{T}}.$$

The gradient of $J_1(\boldsymbol{\theta},\omega)$ is given by

$$\mathrm{grad}[J_1(\boldsymbol{\theta},\omega)] = \begin{bmatrix} \dfrac{\partial J_1(\boldsymbol{\theta},\omega)}{\partial \boldsymbol{\theta}} \\ \dfrac{\partial J_1(\boldsymbol{\theta},\omega)}{\partial \omega} \end{bmatrix} = -2\begin{bmatrix} \boldsymbol{\varphi}(\omega,k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{\theta}] \\ \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}'_{\omega}(\omega,k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{\theta}] \end{bmatrix} \in \mathbb{R}^{2N+1}. \tag{7}$$

Let

$$\boldsymbol{\psi}(\boldsymbol{\theta},\omega,k) := \begin{bmatrix} \boldsymbol{\varphi}(\omega,k) \\ \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}'_{\omega}(\omega,k) \end{bmatrix}.$$

Eq. (7) can be rewritten as

$$\mathrm{grad}[J_1(\boldsymbol{\theta},\omega)] = -2\boldsymbol{\psi}(\boldsymbol{\theta},\omega,k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{\theta}]. \tag{8}$$

Let $\hat{\boldsymbol{\theta}}_k$ and $\hat{\omega}_k$ be the estimates of $\boldsymbol{\theta}$ and ω, respectively. Using the negative gradient search to minimize $J_1(\boldsymbol{\theta},\omega)$, we can obtain the following recursive equation:

$$\begin{aligned} \begin{bmatrix} \hat{\boldsymbol{\theta}}_k \\ \hat{\omega}_k \end{bmatrix} &= \begin{bmatrix} \hat{\boldsymbol{\theta}}_{k-1} \\ \hat{\omega}_{k-1} \end{bmatrix} - \frac{\mu_k}{2}\mathrm{grad}\,[J_1(\boldsymbol{\theta},\omega)]\Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{k-1},\omega=\hat{\omega}_{k-1}} \\ &= \begin{bmatrix} \hat{\boldsymbol{\theta}}_{k-1} \\ \hat{\omega}_{k-1} \end{bmatrix} + \mu_k\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1},\hat{\omega}_{k-1},k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\hat{\boldsymbol{\theta}}_{k-1}], \end{aligned}$$

where μ_k is the step-size to be given later. Let

$$e_k := s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\hat{\boldsymbol{\theta}}_{k-1}; \tag{9}$$

we have

$$\begin{bmatrix} \hat{\boldsymbol{\theta}}_k \\ \hat{\omega}_k \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{k-1} \\ \hat{\omega}_{k-1} \end{bmatrix} + \mu_k\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1},\hat{\omega}_{k-1},k)e_k. \tag{10}$$

The following finds a best step-size μ_k by solving $\min_{\mu_k \geqslant 0} J_1(\hat{\boldsymbol{\theta}}_k,\hat{\omega}_k)$. Let

$$g(\mu_k) := J_1(\hat{\boldsymbol{\theta}}_k,\hat{\omega}_k). \tag{11}$$

Substituting Eq. (10) into Eq. (11), we have

$$\begin{aligned} g(\mu_k) &= \left[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_k,k)\hat{\boldsymbol{\theta}}_k\right]^2 \\ &= \left\{s_k - \boldsymbol{\varphi}^{\mathrm{T}}[\hat{\omega}_{k-1} + \mu_k\hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)e_k,k][\hat{\boldsymbol{\theta}}_{k-1} + \mu_k\boldsymbol{\varphi}(\hat{\omega}_{k-1},k)e_k]\right\}^2. \end{aligned} \tag{12}$$

Let $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$; using the first-order Taylor expansion of $\boldsymbol{\varphi}(\omega,k)$ at $\hat{\omega}_{k-1}$ gives

$$\begin{aligned} g(\mu_k) &= \left\{s_k - [\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k) + [\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)]^{\mathrm{T}}(\hat{\omega}_k - \hat{\omega}_{k-1}) + o(\hat{\omega}_k - \hat{\omega}_{k-1})]\hat{\boldsymbol{\theta}}_k\right\}^2 \\ &= \left\{s_k - [\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k) + [\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)]^{\mathrm{T}}\mu_k\hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)e_k + o(\hat{\omega}_k - \hat{\omega}_{k-1})]\hat{\boldsymbol{\theta}}_k\right\}^2 \\ &= \left\{s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)[\hat{\boldsymbol{\theta}}_{k-1} + \mu_k\boldsymbol{\varphi}(\hat{\omega}_{k-1},k)e_k]\right. \\ &\qquad \left. - [\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)]^{\mathrm{T}}\mu_k\hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)e_k[\hat{\boldsymbol{\theta}}_{k-1} + \mu_k\boldsymbol{\varphi}(\hat{\omega}_{k-1},k)e_k] - o(\hat{\omega}_k - \hat{\omega}_{k-1})\right\}^2 \\ &= \left\{[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\hat{\boldsymbol{\theta}}_{k-1}] - e_k\|\boldsymbol{\varphi}(\hat{\omega}_{k-1},k)\|^2\mu_k - e_k\|\hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)\|^2\mu_k\right. \\ &\qquad \left. - e_k^2\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\hat{\boldsymbol{\theta}}_{k-1}\|\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)\|^2\mu_k^2 - o(\hat{\omega}_k - \hat{\omega}_{k-1})\right\}^2 \\ &= e_k^2\left\{1 - \left[\|\boldsymbol{\varphi}(\hat{\omega}_{k-1},k)\|^2 + \|\hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)\|^2\right]\mu_k\right. \\ &\qquad \left. - e_k\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\hat{\boldsymbol{\theta}}_{k-1}\|\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1},k)\|^2\mu_k^2 - o(\hat{\omega}_k - \hat{\omega}_{k-1})\right\}^2 \\ &= e_k^2\left[1 - \|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1},\hat{\omega}_{k-1},k)\|^2\mu_k - \xi_k\mu_k^2\right]^2 + o(\hat{\omega}_k - \hat{\omega}_{k-1})^2, \end{aligned} \tag{13}$$

where

$$\xi_k := e_k \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1}, k)\hat{\boldsymbol{\theta}}_{k-1}\|\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1}, k)\|^2.$$

Then the optimal μ_k can be obtained by minimizing $g(\mu_k)$. This leads to

$$1 - \|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1}, \hat{\omega}_{k-1}, k)\|^2\mu_k - \xi_k\mu_k^2 = 0. \tag{14}$$

Its solution is

$$\begin{aligned}\mu_k &= \frac{\sqrt{\|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1}, \hat{\omega}_{k-1}, k)\|^4 + 4\xi_k} - \|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1}, \hat{\omega}_{k-1}, k)\|^2}{2\xi_k}\\ &= \frac{2}{\sqrt{\|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1}, \hat{\omega}_{k-1}, k)\|^4 + 4\xi_k} + \|\boldsymbol{\psi}(\hat{\boldsymbol{\theta}}_{k-1}, \hat{\omega}_{k-1}, k)\|^2}.\end{aligned} \tag{15}$$

Eqs. (10), (9) and (15) form the gradient based algorithm for estimating $\boldsymbol{\theta}$ and ω.

4. The least squares based estimation algorithm

Besides the above gradient based method, the unknown parameters $\boldsymbol{\theta}$ and ω in Eq. (5) can also be estimated using the least squares method. According to the least squares principle, define a criterion function:

$$J_2(\boldsymbol{\theta}, \omega) := \frac{1}{2}\sum_{i=1}^{k}[s_i - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\boldsymbol{\theta}]^2. \tag{16}$$

Taylor expansions of $J_2(\boldsymbol{\theta}, \omega)$ at $\hat{\boldsymbol{\theta}}_{k-1}$ and $\hat{\omega}_{k-1}$ are shown as follows:

$$J_2(\boldsymbol{\theta}, \omega) = J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega) + \frac{\partial J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega)}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k-1}) + \frac{1}{2}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k-1})^{\mathrm{T}}\frac{\partial^2 J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega)}{\partial \boldsymbol{\theta}\partial \boldsymbol{\theta}^{\mathrm{T}}}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k-1}) + o(\|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k-1}\|^2), \tag{17}$$

$$J_2(\boldsymbol{\theta}, \omega) = J_2(\boldsymbol{\theta}, \hat{\omega}_{k-1}) + \frac{\partial J_2(\boldsymbol{\theta}, \hat{\omega}_{k-1})}{\partial \omega}(\omega - \hat{\omega}_{k-1}) + \frac{1}{2}\frac{\partial^2 J_2(\boldsymbol{\theta}, \hat{\omega}_{k-1})}{\partial \omega^2}(\omega - \hat{\omega}_{k-1})^2 + o(|\omega - \hat{\omega}_{k-1}|^2). \tag{18}$$

Let

$$\frac{\partial J_2(\hat{\boldsymbol{\theta}}_k, \omega)}{\partial \boldsymbol{\theta}} = \mathbf{0}, \qquad \frac{\partial J_2(\boldsymbol{\theta}, \hat{\omega}_k)}{\partial \omega} = 0, \tag{19}$$

which leads to the corresponding Newton algorithms:

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} - \left[\frac{\partial^2 J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega)}{\partial \boldsymbol{\theta}\partial \boldsymbol{\theta}^{\mathrm{T}}}\right]^{-1}\frac{\partial J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega)}{\partial \boldsymbol{\theta}}, \tag{20}$$

$$\hat{\omega}_k = \hat{\omega}_{k-1} - \left[\frac{\partial^2 J_2(\boldsymbol{\theta}, \hat{\omega}_{k-1})}{\partial \omega^2}\right]^{-1}\frac{\partial J_2(\boldsymbol{\theta}, \hat{\omega}_{k-1})}{\partial \omega}. \tag{21}$$

Using Eq. (16), we have

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \left[\sum_{i=1}^{k}\boldsymbol{\varphi}(\omega, i)\boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\right]^{-1}\sum_{i=1}^{k}\boldsymbol{\varphi}(\omega, i)[s_i - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\hat{\boldsymbol{\theta}}_{k-1}]. \tag{22}$$

Let

$$\boldsymbol{P}_k := \left[\sum_{i=1}^{k}\boldsymbol{\varphi}(\omega, i)\boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\right]^{-1}; \tag{23}$$

then Eq. (22) can be rewritten as

$$\begin{aligned}\hat{\boldsymbol{\theta}}_k &= \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{P}_k\left\{\boldsymbol{\varphi}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\hat{\boldsymbol{\theta}}_{k-1}] + \sum_{i=1}^{k-1}\boldsymbol{\varphi}(\omega, i)[s_i - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\hat{\boldsymbol{\theta}}_{k-1}]\right\}\\ &= \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{P}_k\boldsymbol{\varphi}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\hat{\boldsymbol{\theta}}_{k-1}] + \boldsymbol{P}_k\sum_{i=1}^{k-1}\boldsymbol{\varphi}(\omega, i)[s_i - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, i)\hat{\boldsymbol{\theta}}_{k-1}]\\ &= \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{P}_k\boldsymbol{\varphi}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\hat{\boldsymbol{\theta}}_{k-1}] + \boldsymbol{P}_k\frac{\partial J_2(\hat{\boldsymbol{\theta}}_{k-1}, \omega)}{\partial \boldsymbol{\theta}}\\ &= \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{P}_k\boldsymbol{\varphi}(\omega, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\omega, k)\hat{\boldsymbol{\theta}}_{k-1}].\end{aligned} \tag{24}$$

To avoid computing the inverse matrix $\boldsymbol{P}_k$, applying the formulae [28]

$$(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{C})^{-1}=\boldsymbol{A}^{-1}-\boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1}$$

to Eq. (23) gives

$$\boldsymbol{P}_k=\boldsymbol{P}_{k-1}-\frac{\boldsymbol{P}_{k-1}\boldsymbol{\varphi}(\omega,k)\boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{P}_{k-1}}{1+\boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{P}_{k-1}\boldsymbol{\varphi}(\omega,k)},\qquad \boldsymbol{P}_0=p_0\boldsymbol{I}. \tag{25}$$

Define ϕ as the first-order partial derivative of the criterion function J_2 with respect to ω at time $k-1$:

$$\phi(\boldsymbol{\theta},\omega):=-\sum_{i=1}^{k-1}(\boldsymbol{\varphi}'_\omega)^{\mathrm{T}}(\omega,i)\boldsymbol{\theta}[s_i-\boldsymbol{\varphi}^{\mathrm{T}}(\omega,i)\boldsymbol{\theta}],\qquad \phi(\boldsymbol{\theta},\hat{\omega}_{k-1})=0. \tag{26}$$

Let

$$\varepsilon(\boldsymbol{\theta},\omega):=s_k-\boldsymbol{\varphi}^{\mathrm{T}}(\omega,k)\boldsymbol{\theta}, \tag{27}$$

$$\zeta(\boldsymbol{\theta},\omega):=-\frac{\partial\varepsilon(\boldsymbol{\theta},\omega)}{\partial\omega}=(\boldsymbol{\varphi}'_\omega)^{\mathrm{T}}(\omega,k)\boldsymbol{\theta}, \tag{28}$$

when $\omega=\hat{\omega}_{k-1}$, the first-order partial derivative of J_2 at time k with respect to ω is

$$\begin{aligned}\frac{\partial J_2(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega}&=-\sum_{i=1}^{k}[\boldsymbol{\varphi}'_\omega(\hat{\omega}_{k-1},i)]^{\mathrm{T}}\boldsymbol{\theta}[s_i-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},i)\boldsymbol{\theta}]\\&=-[\boldsymbol{\varphi}'_\omega(\hat{\omega}_{k-1},k)]^{\mathrm{T}}\boldsymbol{\theta}[s_k-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\boldsymbol{\theta}]-\sum_{i=1}^{k-1}[\boldsymbol{\varphi}'_\omega(\hat{\omega}_{k-1},i)]^{\mathrm{T}}\boldsymbol{\theta}[s_i-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},i)\boldsymbol{\theta}]\\&=-\zeta(\boldsymbol{\theta},\hat{\omega}_{k-1})\varepsilon(\boldsymbol{\theta},\hat{\omega}_{k-1})+\phi(\boldsymbol{\theta},\hat{\omega}_{k-1})\\&=-\zeta(\boldsymbol{\theta},\hat{\omega}_{k-1})\varepsilon(\boldsymbol{\theta},\hat{\omega}_{k-1}).\end{aligned} \tag{29}$$

The second-order partial derivative of J_2 at time k when $\omega=\hat{\omega}_{k-1}$ can be given as

$$\frac{\partial J_2^2(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega^2}=\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega}+\zeta^2(\boldsymbol{\theta},\hat{\omega}_{k-1})+\frac{\partial^2\varepsilon(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega^2}\varepsilon(\boldsymbol{\theta},\hat{\omega}_{k-1}). \tag{30}$$

Neglecting the last item, Eq. (30) is reduced to

$$\frac{\partial J_2^2(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega^2}=\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega}+\zeta^2(\boldsymbol{\theta},\hat{\omega}_{k-1}). \tag{31}$$

Let

$$R_k:=\frac{\partial J_2^2(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega^2}=\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega}+\zeta^2(\boldsymbol{\theta},\hat{\omega}_{k-1}). \tag{32}$$

Obviously R_{k-1} is the second-order partial derivative of J_2 at time $k-1$ when $\omega=\hat{\omega}_{k-2}$; according to the definition of ϕ in Eq. (26), R_{k-1} can be expressed as

$$R_{k-1}=\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-2})}{\partial\omega}. \tag{33}$$

Approximating $\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-1})}{\partial\omega}$ by $\frac{\partial\phi(\boldsymbol{\theta},\hat{\omega}_{k-2})}{\partial\omega}$ in (32) and using (33), we have

$$R_k=R_{k-1}+\zeta^2(\boldsymbol{\theta},\hat{\omega}_{k-1})=R_{k-1}+\{[\boldsymbol{\varphi}'_\omega(\hat{\omega}_{k-1},k)]^{\mathrm{T}}\boldsymbol{\theta}\}^2. \tag{34}$$

Substituting (29) and (32) into (21), the iterative algorithm of ω can be achieved:

$$\begin{aligned}\hat{\omega}_k&=\hat{\omega}_{k-1}+R_k^{-1}\zeta(\boldsymbol{\theta},\hat{\omega}_{k-1})\varepsilon(\boldsymbol{\theta},\hat{\omega}_{k-1})\\&=\hat{\omega}_{k-1}+R_k^{-1}[\boldsymbol{\varphi}'_\omega(\hat{\omega}_{k-1},k)]^{\mathrm{T}}\boldsymbol{\theta}[s_k-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1},k)\boldsymbol{\theta}].\end{aligned} \tag{35}$$

However, the above algorithm in (24) and (25) or in (34) and (35) is technically impossible to be implemented since each of them contains an unknown parameter or parameter vector, i.e. ω or $\boldsymbol{\theta}$. To solve this difficulty, we use the hierarchical

Table 1
The parameter and frequency estimates and errors.

k	a_1	b_1	a_2	b_2	ω	δ (%)
1	68.40651	88.48776	−27.16348	107.79215	319.15926	59.61121
2	102.97164	57.40956	61.10889	134.71411	319.16482	54.36552
5	260.92582	65.65123	36.14786	21.64655	319.19376	11.68815
10	299.12142	47.13965	19.17972	8.25504	319.22128	2.06155
20	297.93661	47.21945	19.26988	8.92053	319.13759	2.08395
30	298.54934	47.51381	19.65887	8.59253	317.68829	1.95755
True values	300.00000	40.00000	20.00000	10.00000	314.15927	

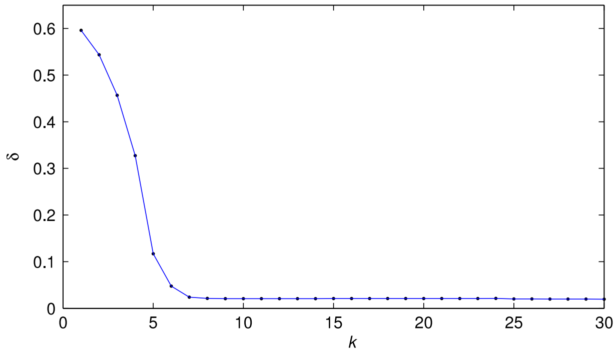

Fig. 1. The estimation error δ versus k.

identification principle [29–31]: replacing ω in (24) and (25) with $\hat{\omega}_{k-1}$ and $\boldsymbol{\theta}$ in (34) and (35) with $\hat{\boldsymbol{\theta}}_{k-1}$ leads to the least squares based algorithm for estimating $\boldsymbol{\theta}$ and ω:

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{P}_k \boldsymbol{\varphi}(\hat{\omega}_{k-1}, k)[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1}, k)\hat{\boldsymbol{\theta}}_{k-1}], \tag{36}$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k-1} - \frac{\boldsymbol{P}_{k-1}\boldsymbol{\varphi}(\hat{\omega}_{k-1}, k)\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1}, k)\boldsymbol{P}_{k-1}}{1 + \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1}, k)\boldsymbol{P}_{k-1}\boldsymbol{\varphi}(\hat{\omega}_{k-1}, k)}, \qquad \boldsymbol{P}_0 = p_0 \boldsymbol{I}, \tag{37}$$

$$\hat{\omega}_k = \hat{\omega}_{k-1} + R_k^{-1}[\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1}, k)]^{\mathrm{T}}\hat{\boldsymbol{\theta}}_{k-1}[s_k - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\omega}_{k-1}, k)\hat{\boldsymbol{\theta}}_{k-1}], \tag{38}$$

$$R_k = \lambda R_{k-1} + \{[\boldsymbol{\varphi}'_{\omega}(\hat{\omega}_{k-1}, k)]^{\mathrm{T}}\hat{\boldsymbol{\theta}}_{k-1}\}^2, \quad 0 \leqslant \lambda \leqslant 1. \tag{39}$$

In this algorithm, we have introduced a forgetting factor λ in (39). The initial value $\hat{\boldsymbol{\theta}}_0$ is taken to be a random vector, $\hat{\omega}_0$ to be a random number, $\boldsymbol{P}_0 = p_0\boldsymbol{I}$, $p_0 = 10^6$ and $R_0 = 1$.

5. Example

Consider the following corrupted voltage signal:

$$y(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t) + v(t),$$

where $\omega = 2\pi f$, $f = 50$ Hz is the fundamental frequency, and the amplitudes of the harmonic waves are $a_1 = 300$, $b_1 = 40$, $a_2 = 20$ and $b_2 = 10$.

In simulation, $\{v(t)\}$ is taken as a white noise process with zero mean and variance $\sigma^2 = 0.50^2$. Using the measured discrete samples $\{s_k, s = 0, 1, 2, \ldots\}$, applying the proposed least squares based algorithm in (36)–(39) with $\lambda = 0.20$ to estimate parameters a_i, b_i and frequency ω, the parameter and frequency estimates are shown in Table 1, where the estimation error

$$\delta := \sqrt{\frac{\|\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}\|^2 + (\hat{\omega}_k - \omega)^2}{\|\boldsymbol{\theta}\|^2 + \omega^2}}$$

versus k is shown in Fig. 1, and the actual signal $s(t)$ and estimated signal $\hat{s}(t)$ versus t are shown in Fig. 2.

From Table 1 and Figs. 1 and 2, we can draw the following conclusions.

1. The estimation error becomes small with the data length k increasing — see Table 1 and Fig. 1.
2. The estimated signal $\hat{s}(t)$ can track the actual signal $s(t)$ — see Fig. 2.

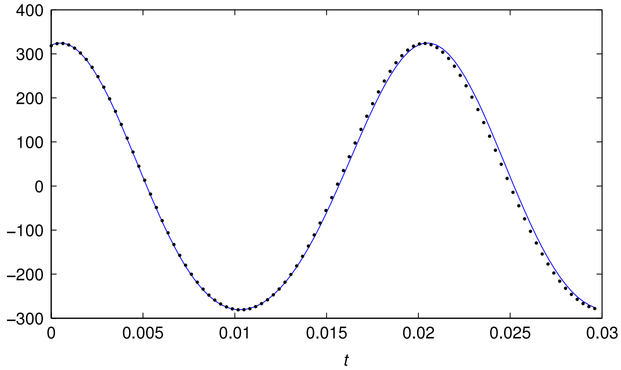

Fig. 2. The actual signal $s(t)$ and estimated signal $\hat{s}(t)$ versus t.

6. Conclusions

In this paper, a gradient based algorithm and a least squares based algorithm are derived to model periodic signals from a power system. According to the simulation with a distorted sinusoidal signal, the fundamental frequency and the harmonic parameters can be accurately estimated with the hierarchical identification principle.

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