# Local Energy based Image Fusion in Sharp Frequency Localized Contourlet Transform

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## Abstract

Image fusion method based on multiscale transform (MST) is a popular choice in recent research. Sharp frequency localized contourlet transform (SFLCT) that significantly outperform the original contourlet transform is proposed. Commonly, the upsamplers and the downsamplers presented in directional filter banks of SFLCT make the resulting image not shift-invariant and easily cause the pseudo-Gibbs phenomena. In order to suppress the pseudo-Gibbs phenomena, we apply cycle spinning as compensation. Then, the coefficients of shifted images are calculated. We take the following image fusion rules. First, cycle spinning the source images, the shifted images are obtained. Second, selecting the low-frequency coefficients by the local energy method and calculating the high-frequency coefficients by the sum modified Laplacian (SML), and the coefficients fusion follows. Third, applying the inverse SFLCT and the inverse cycle-spinning sequentially, the image is reconstructed. Numerical experiment results show that the proposed method significantly outperform the wavelet transform, the pyramid transform and the curvelet transform both in visual quality and in quantitative analysis.

*Keywords:* Image Fusion; Centroid Sharp Frequency Localized Contourlet Transform (SFLCT); Directional Filter Banks; Cycle Spinning; Local Energy; Sum Modified Laplacian (SML)

## 1. Introduction

For all visible-light imaging system, due to the limited scope of focus imaging system, it is difficult to display all of the goals clearly. This problem can be solved by multi-focus image fusion techniques. That is, use the same imaging lens on the targets twice or more and fuse the clear part of these images into a new image in order to facilitate human observation or computer processing. This technique can be applied to remote sensing, medical image processing, high-definition digital TV and so on.

At present, there are some commonly used fusion algorithms such as weighted average method [1], pyramidal algorithm [2], wavelet transform method [3] and curvelet transform method [4] and contourlet transform [5]. The weighted average method is one of the simplest image fusion methods. The source images do not be transformed and decomposed and fused image directly averages the gray level of defocused images' pixels. This method is suitable for real-time processing, but will decrease the signal to noise ratio of the image. The pyramid method firstly constructs the input image pyramid, and then takes some feature selection approach to form the fusion value pyramid. Through the inverter of the pyramid, the

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pyramid of image can be reconstructed, to produce fusion images. This method is relatively simple, but it also has some drawbacks. We can see these in [5]. Using wavelet transform method, the image can be decomposed into a series of sub-band images with different resolution, frequency and direction characteristics. The spectral characteristics and spatial characteristics of image are completely separated. And then the different resolution image fusion is gotten. Minh N. Do and Martin Vetterli proposed contourlet transform [6]. That first develops a transform in the continuous domain and then discretize for sampled data. After that, Yue Lu and Minh N. Do [7] modified a new multiscale decomposition method in the frequency domain. However, due to upsamplers and downsamplers presented in the directional filter banks (DFB) [8] of sharp frequency localization contourlet transform (SFLCT), SFLCT is not shift-invariant, and easily causes pseudo-Gibbs phenomena. Qu Xiao-bo proposed a method can limit this flaw, which is called CS-SFLCT [9].

In this paper, we apply cycle spinning sharp frequency localization contourlet transform (CS-SFLCT) to image fusion. Particularly, for multi-focus image fusion, we selected the low-frequency coefficients by local energy (LE) method [10], and introduced sum modified Laplacian (SML) [11] to calculate the high-frequency coefficients. In Section 2 briefly introduces contourlet, sharp frequency localization contourlet transform. As a solution, we propose in Section 3 a new method to fusion. The experiments are presented in Section 4 to confirm our method is a better one. Section 5 concludes the paper.

## 2. Background and Related Work

The original contourlet is constructed by the combination of Laplacian pyramid [6]. The Laplacian pyramid shown in the diagram is a simplified version of its actual implementation as shown in Fig. 1.
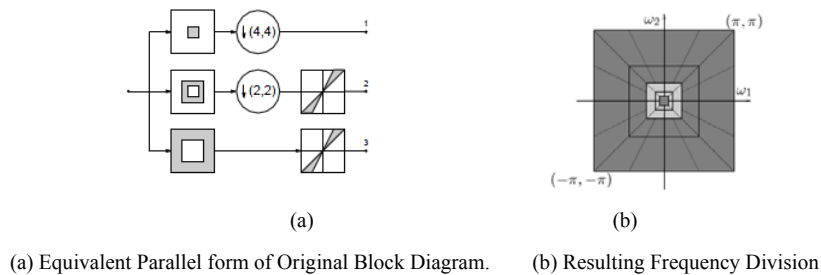


(a)                                      (b)

(a) Equivalent Parallel form of Original Block Diagram.        (b) Resulting Frequency Division

Fig.1 The Original Contourlet Transform



(a)                    (b)                    (c)                    (d)
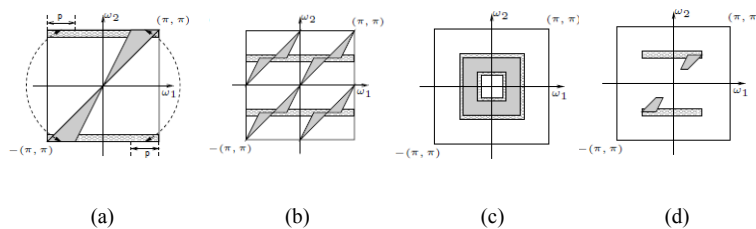
Fig. 2 Illustration of the Frequency Domain Aliasing Problem of the Contourlet Transforms. (a) One Directional Filter. (b) The Directional Filter after being upSampled by 2 along Each Dimension. (c) A Bandpass Filter from the Laplacian Pyramid. (d) The Resulting Contourlet Subband.

However, the frequency division in Fig. 1(b) is obtained by ideal filters. When non-ideal filters are combined with Laplacian pyramid, we show a more realistic illustration of one of the directional filters from the direction filter banks in Fig. 2(a). If the directional filter must first be upsampled by 2 along each dimensions, which as shown in Fig. 2(b). Because of the upsampling, the aliasing components are folded towards the lowpass regions and concentrated mostly along two lines $\omega_2 = \pm \pi/2$. Combining the upsampled DFB was shown in Fig. 2(c). In Fig. 2(d), we can see the resulting contourlet subband.

In order to solve this problem, Yue M. Lu proposed a new construction of a sharp frequency localization contourlet transform (SFLCT) [7]. Instead of using the Laplacian pyramid, he employed a new pyramid structure for multiscale decomposition, which is shown in Fig. 3.
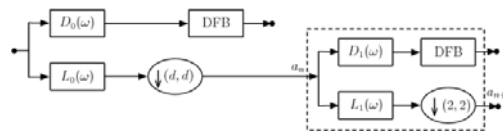


Fig. 3 The Block Diagram of SF.

The difference between SFLCT and contourlet transform is that, SFLCT use the new multiscale pyramid and can employ a different set of lowpass and highpass filters for the levels. Suppose lowpass filters $L_i(\omega)$ ($i$ = 0,1) in the frequency domain as $L_i(\omega) = L_i^{ld}(\omega_1) \cdot L_i^{ld}(\omega_2)$, and $L_i^{ld}(\omega)$ is a one-dimensional lowpass filter with passband frequency $\omega_{p,i}$ and stopband frequency $\omega_{s,i}$ and a smooth transition band, defined as

$$L_i^{ld}(\omega) = \begin{cases} 1 & for |\omega| \leq \omega_{p,i}, \\ \dfrac{1}{2} + \dfrac{1}{2}\cos\dfrac{(|\omega| - \omega_{p,i})\pi}{\omega_{s,i} - \omega_{p,i}} & for \omega_{p,i} < |\omega| < \omega_{s,i} \\ 0 & for \omega_{s,i} \leq |\omega| \leq \pi \end{cases} \tag{1}$$

for $|\omega| \leq \pi$ and $i = 0,1$.

Under the assumption that aliasing can be completely cancelled, the perfect reconstruction condition for the multiscale pyramid should be satisfied with

$$|L_i(\omega)|^2 + |D_i(\omega)|^2 \equiv 1, \quad for \ i = 0, 1 \dots \tag{2}$$

Fig. 4 shows the comparison on basis image of the original contourlet and SFLCT.
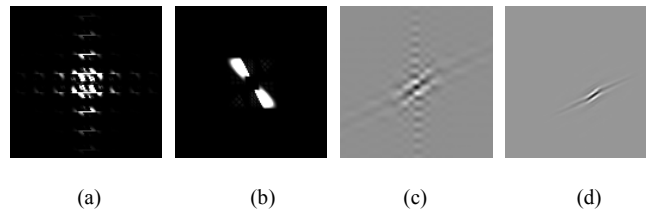


|   (a)   |   (b)   |   (c)   |   (d)   |

Fig. 4 Comparison on Basis image of the Original Contourlet and SFLCT. (a) and (b) Basis Images of Original Contourlet and Sharp Frequency Localized Contourlet in Frequency Domain. (c) and (d) Basis Image of the Two Transforms in Spatial Domain.

## 3. Proposed Image Fusion Method

### 3.1. Cycle Spinning

Because of downsamplers and upsamplers used in directional filter banks of SFLCT, it is not shift-invariant, which easily causes pseudo-Gibbs phenomena around singularities and is affected the results of multifocus image fusion. Cycle spinning (CS) [12] is introduced to estimate the drawback.

Suppose $f_1$, $f_2$ are source images and $F$ is the fused image, $C_{-1}, C$ are the inverse SFLCT and forward SFLCT, $S_{x,y}$ is the cycle spinning method and $x,y$ are the shift arranges in horizontal and vertical directions. Fusion method is

$$F = S_{-x,-y}\{h[C(\ S_{x,y}(f_1)\ ),\ C(\ S_{x,y}(f_2)\ )]\}. \tag{3}$$

where, $h$ is the function process in SFLCT domain. $x \in X$ and $y \in Y$ is the shift arranges, $X = \{x_1, x_2, ..., x_m\}$, $Y = \{y_1, y_2, ..., y_n\}$.

Therefore, cycle spinning averages the dependence of directional filter banks of SFLCT. It can be defined as

$$F = Ave_{x \in X, y \in Y}\left\{ S_{-x,-y}\left\{ h\left[ C\left( S_{x \in X, y \in Y}(f_1) \right), C\left( S_{x \in X, y \in Y}(f_2) \right) \right] \right\} \right\}. \tag{4}$$

By changing the arrangement sequence of the image, cycle spinning changes the singular point of the image, in order to reduce or eliminate the oscillation amplitude and to improve the reconstruction quality.

### 3.2. Local Energy

After Fourier transform into the frequency domain, there are high frequency and low frequency of sub. Most of the energy of image concentrated in low frequency coefficients. It is greatest impact the image quality. So, how to choose the low frequency coefficients is the key to improve the image quality.

All kinds of beyond wavelet transforms are based on the geometric features of image analysis method, to achieve multi-scale and multi-directional image decomposition. This is suitable for the line singular analysis. But because these transforms contain dowmsampling, it is not translation invariance, leading to pseudo-Gibbs effects. In other side, as the incomplete of the multi-scale decomposition, some details of the image are still remaining in the low frequency components. This phenomenon is obviously when the decomposition levels are less. Because of this, someone suggested that use edge-based fusion method in low frequency.

In this study, we use the local energy (LE) [10] as a measurement to choose the low frequency coefficients. Select the maximum energy of two source images as output. Due to the partial human visual perception characteristics and the relationship of decomposition about local correlation coefficients, the statistical characteristics of neighbor should be considered. Therefore, the statistic algorithm is based on the $3 \times 3$ window. The algorithm is described as follows:

$$LE_\xi(i,j) = \sum_{i \in M, j \in N} p(i+i', j+j') \cdot f_\xi^{(0)2}(i+i', j+j'). \tag{5}$$

where $p$ is the local filtering operator. $M, N$ is the scope of local window. $\xi \in A$ or $B$ ($A$, $B$ is the window for scanning two images). $f_\xi^{(0)}(i,j)$ is low frequency coefficients.

Local contourlet energy is

$$LCE_{\xi}^{l,k}(i,j) = E_1 * f_{\xi}^{(0)2}(i,j) + E_2 * f_{\xi}^{(0)2}(i,j) + \cdots + E_K * f_{\xi}^{(0)2}(i,j). \tag{6}$$

where $E_1, E_2, \ldots, E_{K-1}$ and $E_K$ are the filter operators in $K$ different directions. In this paper, we use 3 directional filtering operators extract the edge information with spatial filters for low frequency.

$$E_1 = \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad E_2 = \begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix} \quad E_3 = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

Suppose $I_A^{l,k}(i,j)$, $I_B^{l,k}(i,j)$ and $I_F^{l,k}(i,j)$ denote the coefficients of source images and fused images. The proposed LE-based fusion rule can be described as follows.

### 3.3. Sum Modified Laplacian

Under the assumption that image details are contained in the high-frequency subbands in Multi-scale domain, the typical fusion rule is maximum-based rule, which selects high-frequency coefficients with maximum absolute value. Recently, there are many measurements, such as energy of gradient (EOG), spatial frequency (SF), Tenengrad, energy of laplace (EOL) and sum modified laplacian (SML). In this paper, we use SML for choosing the high frequency coefficients.

At the same time, a focus measure is defined in a maximum for the focused image. Therefore, for multifocus image fusion, the focused image areas of the source images must produce maximum focus measures. Set $f(x,y)$ be the gray level intensity of pixel $(x,y)$. Defined modified Laplacian (ML) [9] [11] is

$$\nabla_{ML}^2 f(x,y) = \left| 2f(x,y) - f(x-\text{step},y) - f(x+\text{step},y) \right| \\ + \left| 2f(x,y) - f(x,y-\text{step}) - f(x,y+\text{step}) \right| \tag{8}$$

In this paper "step" always equals to 1.

$$SML_x^{l,k}(i,j) = \sum_{i=-M}^{M} \sum_{j=-N}^{N} \nabla_{ML}^2 f(i+p,j+q) \quad for \quad \nabla_{ML}^2 f(i,j) \geq T \tag{9}$$

where, $l, k$ respectively the scale and the direction of transform. $x \in A$ or $B$ is respectively the source images. $T$ is a discrimination threshold value. $M, N$ determine the window with size of $(2M+1) \times (2N+1)$.

Suppose $C_A^{l,k}(i,j)$, $C_B^{l,k}(i,j)$ and $C_F^{l,k}(i,j)$ denote the coefficients of source images and fused images. The proposed SML-based fusion rule can be described as follows:

$$C_F^{l,k}(i,j) = \begin{cases} C_A^{l,k}(i,j), if : SML_A^{l,k}(i,j) \geq SML_B^{l,k}(i,j) \\ C_B^{l,k}(i,j), if : SML_A^{l,k}(i,j) < SML_B^{l,k}(i,j) \end{cases}. \tag{10}$$

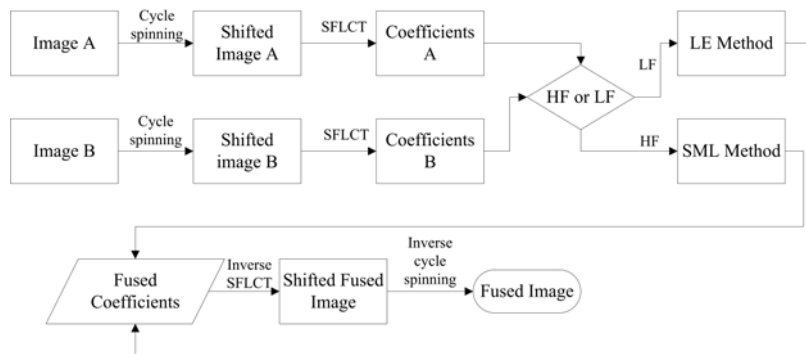The all progress of fusion can be expressed as Fig. 5.

Fig. 5 Framework of Image Fusion Method in Our Paper.

## 4. Experiments

In this section, the resulting images of the proposed "CS-SFLCT-LE-SML" method and several other approaches are shown for evaluation. From the Fig. 6(c)-(g), we can see that the wavelet transform method and curvelet transform method give a clear panoramic image. However, comparing with the wavelet transform method and curvelet method, our method obtains a clearer image. The details are more prominent, the texture is clearer, and the blurring elimination is more effective. Fig. 7 shows the enlarged result images of CS-SFLCT-SML transform and CS-SFLCT-LE-SML transform. We can clearly see that the corner of the clock (b) is more smooth and clearly than that of clock (a).
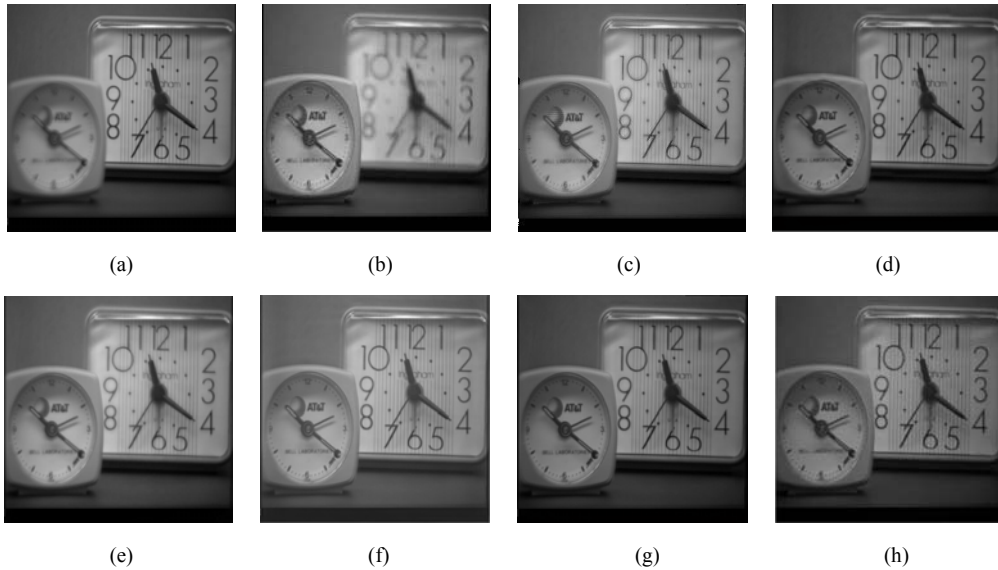


Fig.6 Different results of Different Transforms by Visual Observation. (a) Right Focus Image. (b) Left Focus Image. (c) Result of Median Pyramid Transform. (d) Result of Wavelet Transform. (e) Result of Curvelet Transform. (f) Result of LE-curvelet Transform.

(g) Result of CS-SFLCT-SML Transform. (h) Result of CS-SFLCT-LE-SML Method

In addition to visual analysis, we conducted some quantitative analysis, mainly from the perspective of which include entropy, standard deviation, average gradient and cross entropy [13] and so on.

The entropy criterion measures the information content in an image. An image with high information content will have a high entropy. The computational formula is given by

$$H = -\sum_{l=0}^{L-1} p_F(l) \ln p_F(l). \tag{11}$$

where $l \in \{0,1,2,\cdots,L-1\}$, $p_F(l)$ is the probability of fused image $F$ at gray-level $l$.

Standard deviation is the second measurement index. The standard deviation criterion measures the contrast in an image; an image with a high contrast will have a high standard deviation. It is defined as

$$\sigma = \sqrt{\sum_{g=0}^{L-1}(g-\bar{g})^2 p(g)} \tag{12}$$

where L is the number of gray levels in the image, g is the gray level value, $\bar{g} = \sum_{g=0}^{L-1} g \cdot p(g)$ is the average gray value, and $p(g) = \dfrac{number\ of\ pixels\ with\ value\ g}{total\ number\ of\ pixels}$ is the probability that a pixel has a value g.

The average gradient reflects the small details of the image, texture variation and clarity. If this value is larger, the fused image better. It is defined by

$$g = \frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} \left[ (\Delta F_x^{\,2} + \Delta F_y^{\,2})/2 \right]^{\frac{1}{2}} \tag{13}$$

where, $\Delta F_x = F(x, y+1) - F(x, y), \Delta F_y = F(x+1, y) - F(x, y)$.
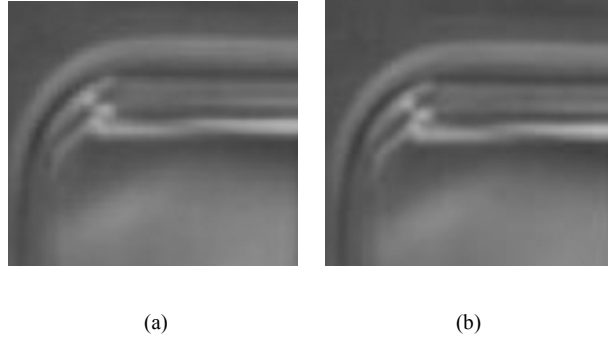


(a)                       (b)

Fig.7 Enlarged Image between CS-SFLCT-SML Transform and CS-SFLCT-LE-SML Transform. (a) is Enlarged Image of CS-SFLCT-SML Transform and (b) is Enlarged Image of CS-SFLCT-LE-SML Transform

Cross entropy evaluates the similarity in information content between images; images containing approximately the same information will have a low cross entropy. We suppose $X$ and $F$ are the original image and fused image. Then

$$CE_{(X,F)} = \sum_{l=0}^{L-1} p_X(l) \ln \frac{p_X(l)}{p_F(l)}. \tag{14}$$

where $X \in A$ or $B$, $A$ and $B$ are source images. $p_F(l)$, $p_X(l)$ is the probability of fused image $F$ and source image $X$ at gray-level $l$. And $N$ source images average cross entropy is

$$\overline{CE}_{(X,F)} = \frac{1}{N} \sum_{1}^{N} CE_{(X,F)}. \tag{15}$$

It measures the difference between the fused image and the source images. If the value of cross entropy is small, the fused image extracts more information from the source images.

Generally, our method improved some of performance parameters. We can see from above table that the parameters of our method are better than those of other methods. Moreover, the subjective visual assessment of the results is consistent to the quantitative analysis.

Table 1 Quantitative Analysis

| Methods | Entropy | Standard Deviation | Average Gradient | Cross Entropy |
|---|---|---|---|---|
| Median Pyramid | 6.7871 | 133.47 | 1.9786 | 11.015 |
| Wavelet Transform | 7.0425 | 89.537 | 2.8884 | 11.396 |
| Curvelet Transform | 7.4050 | 115.20 | 3.2950 | 11.544 |
| LE-curvelet Transform | 7.1056 | 119.63 | 3.6573 | 11.068 |
| CS-SFLCT-SML | 5.6125 | 167.14 | 2.6827 | 12.525 |
| CS-SFLCT-LE-SML | 5.6207 | 182.91 | 2.6932 | 11.063 |

From the Table 1, the entropy of median pyramid, wavelet transform, curvelet transform and LE-curvelet transform is larger than that of CS-SFLCT-SML and CS-SFLCT-LE-SML. This is because the downsamplers and upsamplers in multi-scale analysis is not shift-invariant, and easily causes pseudo-Gibbs

phenomena around singularities. And then, the probability of the fused image is changed. So, this causes the entropy larger. From the second measurement parameter, we can found that the local energy method highly improved the quality of the fused image. Compare the average gradient between CS-SFLCT-SML and CS-SFLCT-LE-SML, the numerical of the later is a little better than the former. We also can found that the average gradient of LE-curvelet transform is obviously improved than the original curvelet method. The cross entropy of CS-SFLCT-LE-SML is smaller than CS-SFLCT-SML, which certify that the local energy is an appropriate method for image fusion. In short, the proposed local-energy gives a good result.

## 5.  Conclusions and Future Work

In this paper, a new fusion method for multi-focus images was proposed. Comparing with other methods, the new method produces fused image with better performance. In contrast to wavelet transform and curvelet transform, it exhibits high directional sensitivity and high anisotropic characteristic. The measurement parameters show that the proposed CS-SFLCT-LE-SML obtains better results than that of wavelet transform, median pyramid transform and curvelet transform.

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## References

[1]   ZHANG Li, CHANG Ben-kang, YU Song-tao. Fusion arithmetic and result analysis of CT-MRI images [J]. Chinese Medical Equipment Journal. 2006, 27(10): 5-6.
[2]   HU Xuelong, and SHEN Jie. An algorithm on image fusion based on median pyramid [J]. Microelectronics & Computer. 2008, 25(9):165-165.
[3]   HU Xuelong, and SHEN Jie. Algorithm on Multiresolution Remote Image Fusion with Watershed Segmentation [J]. Journal of Image and Graphics,2010, 15(5):820-823.
[4]   YANG Jun, and ZHAO Zhong-ming. Multi-focus image fusion method based on curvelet transform [J]. Opto Electronic Engineering, 2007, 34(6):67-71.
[5]   Jie SHEN, and Xuelong HU. Image Fusion Algorithm Based on the Nonsubsampled Contourlet Transform [J]. Microelectronics & Computer. 2008, 26(10):74-77, 81.
[6]   M.N.Do and M.Vetterli. The contourlet transform: an efficient directional multiresolution image representation [J]. IEEE Trans. Image Proc., 2005, 14(12)
[7]   Yue Lu and M.N.Do. A new contourlet transform with sharp frequency localization [C]. International Conference on Image Processing, IEEE, Atlanta, USA, pp.1629-1632, 2006.
[8]   R.H. Bamberger and M.J.T. Smith. A filter bank for the directional decomposition of images: theory and design [J]. IEEE Trans. Signal Proc., 1992, 40(4):882-893.
[9]   QU Xiao-bo, YAN Jing-wen, YANG Gui-de. Sum modified laplacian based multifocus image fusion method in sharp frequency localized contourlet transform domain. Optics and Precision Engineering, 2009, 17(5): 1204-1212.
[10]  WANG Jie-xiong, QI Xiang-yang, and LI Xin-wei. Edge and local energy NSCT based remote sensing image fusion. Journal of the Graduate School of the Chinese Academy of Sciences, 2009, 26(5).

[11] J. Yang, Y. Ma, W. Yao, and W.T.Lu, A spatial domain and frequency domain integrated approach to fusion multifocus images," The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences. B7-3, pp. 1147-1151, 2008.

[12] Eslami R, Radha H. The contourlet transform for image denoising using cycle spinning [C]. Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, USA, pp.1982-1986, 2003.

[13] WANG Run-sheng. Information Fusion [M]", Beijing: Science Press, pp.161-165, 2007.