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Journal of Vibration and Control 2007; 13; 29
DOI: 10.1177/1077546306068058

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(Received 16 March 2004; accepted 2 May 2006)

Abstract: Viscoelastic (VE) dampers are one of the most common earthquake mitigation devices. This paper addresses the mathematical modelling of VE dampers and the dynamic analysis of structures with VE dampers. In this paper, the equivalent standard solid model, a new mathematical model of VE dampers, is used to describe the influence of temperature on the energy absorption features of VE dampers. Elastoplastic time field analysis, frequency field analysis and shaking table tests are used to analyze responses of a 1/5-scale three-story reinforced concrete frame structure with and without VE dampers. Comparisons between the numerical and experimental results show that the VE dampers can be modeled by the equivalent standard solid model and that the VE dampers are effective in reducing the seismic responses of structures.

Keywords: Viscoelastic dampers, shaking table test; dynamic response; earthquake mitigation

1. INTRODUCTION

Improving the earthquake-resistance of structures using a variety of mitigation devices has received considerable attention in recent years. Among the available devices, viscoelastic (VE) dampers are considered to be an ideal shock absorption device because of their cost-effectiveness and high reliability. Many studies and tests on VE dampers (see, for example, Chang et al. (1992), Inaudi (1996), Kasia et al. (1993), Lee et al. (2002), Shen and Soong (1995), Tsai (1994) and Xu (2001)) have shown that the energy absorption properties are dependent on the ambient temperature, excitation frequency and strain amplitude. Several mathematical models, such as the Kelvin model, Maxwell model, standard linear solid model, complex parameter model, four parameters model and fractional derivative model, have been proposed for reproducing the experimental behaviour of VE dampers. Of these, only the fractional derivative model can reflect the influence of temperature on VE dampers; it is, however, so complex that it is difficult to apply in structural analysis. In order to verify the effectiveness of VE dampers, many shaking table tests on structures with VE dampers have been carried out (Chang et al., 1991; Chang et al., 1996; Foutch et al., 1993; Lee et al., 1990; Lin et al., 1991; Lobo et al., 1993; Ou and Zou, 1999). Most of test models were steel structures, however, and there is little experimental information on reinforced concrete structures.

Journal of Vibration and Control, **13(1):** 29–43, 2007 DOI: 10.1177/1077546306068058

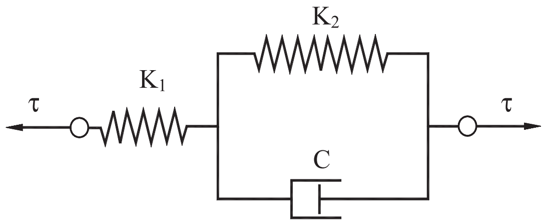

Figure 1. The standard linear solid model.

This paper presents a new mathematical model (the equivalent standard solid model) for VE dampers, which can describe the influence of temperature on the energy absorption features of VE dampers. Shaking table tests on 1/5-scale three-story reinforced concrete frame structures both with and without VE dampers were carried out. Then elastoplastic analysis of the test frame with VE dampers was performed, in which the structure was modeled using the trilinear stiffness degeneration model and the damper was modeled using the equivalent standard solid model. Experimental and analytical results show that VE dampers are effective in reducing the dynamic responses of, and damage to, structures. It is also shown that nonlinear analysis of structures with VE dampers is acceptably accurate when the damper is modeled by the equivalent standard solid model.

2. THE EQUIVALENT STANDARD SOLID MODEL

Temperature and frequency are the main factors that affect the energy dissipation property of VE dampers. Previous research (Chang et al., 1992; Inaudi, 1996; Kasia et al., 1993; Tsai, 1994) has shown that the storage modulus G_1 (which reflects the VE damper's stiffness) decreases with increasing temperature and increases with increasing frequency, and the loss factor η (which reflects the VE damper's energy dissipation capacity) has an optimum value which varies with temperature and frequency. The Kelvin model, Maxwell model and standard linear solid model are usually used to simulate VE dampers' energy dissipation property. The Kelvin model of VE dampers consists of a linear spring in parallel with a viscous element, and the Maxwell model consists of a linear spring in series with a viscous element. Details of the two models are explained in the literature (Shen and Soong, 1995; Xu, 2001; Zhou and Liu, 1996), and thus they are not discussed in detail here.

2.1. The Standard Linear Solid Model

The standard linear solid model of VE dampers consists of a linear spring in series with the Kelvin model, as shown in Figure 1. The relationship between the stress and the strain is given by

$$\tau + p_1\dot{\tau} = q_0\gamma + q_1\dot{\gamma} \tag{1}$$

where q_0, q_1 and p_1 are coefficients related to the VE material, and τ and γ are the shear stress and shear strain, respectively. For harmonic deformation, if the Fourier transformation is applied to equation (1), the following equations can be obtained

$$\left.\begin{aligned} G_1 &= \left(q_0 + p_1 q_1 \omega^2\right) / \left(1 + p_1^2 \omega^2\right) \\ \eta &= (q_1 - p_1 q_0)\omega / \left(q_0 + p_1 q_1 \omega^2\right) \end{aligned}\right\} \tag{2}$$

where ω is frequency. When the frequency ω increases, the storage modulus G_1 can be increased by adjusting coefficients q_0, q_1 and p_1. It can be seen from equation (2) that the loss factor η will reach a maximum value at a fixed frequency. These characteristics reflect the frequency's influence on the behavior of VE dampers, but this model cannot account for the effects of temperature on the behavior of VE dampers.

2.2. Temperature-frequency Equivalence Theory

The storage modulus G_1 and the loss factor η are functions of the temperature T and the frequency ω. Studies such as those of Zhou and Liu (1996) and Zhang (1990) show that the temperature effect is similar to the effects of frequency, provided that the ambient temperature of the VE dampers is between their glass temperature T_g and $T_g + 100°\mathrm{C}$, but inverse (i.e. a low temperature has an effect equivalent to that of a high frequency, and a high temperature's effect is equivalent to a low frequency's effect). The effects of temperature and frequency on VE dampers can be considered together if the storage modulus G_1 and the loss factor η are expressed as

$$\left.\begin{aligned} G_1(\omega, T) &= G_1(\alpha_T \omega, T_0) \\ \eta(\omega, T) &= \eta(\alpha_T \omega, T_0) \end{aligned}\right\} \tag{3}$$

where T_0 is the reference temperature, and α_T is the temperature transformation coefficient, which can be described by the following empirical formula (Zhang, 1990)

$$\alpha_T = 10^{-12(T-T_0)/[525+(T-T_0)]}. \tag{4}$$

2.3. The Equivalent Standard Solid Model

In order to reflect the effects of temperature and frequency on VE dampers, the temperature-frequency equivalence theory is applied to the standard linear solid model, and the equivalent standard solid model is thus created.

The temperature and frequency ranges for VE dampers used in civil engineering are $-30°\mathrm{C} \le T \le 60°\mathrm{C}$ and $0.1\mathrm{HZ} \le \omega \le 10\mathrm{HZ}$, respectively (Xu, 2001). Within these ranges, the frequency ω in equation (2) will be changed into the transformed frequency $\alpha_T \omega$ and the index of frequency will be altered so as to describe the effects of both the frequency and the temperature on the parameters of the VE dampers. The storage modulus G_1 and loss factor η can be written as

Table 1. Comparison between experimental and analytical results.

		Test tesults		Calculated results	
T	ω	G_1 (Mpa)	η	G_1 (Mpa)	η
−20	1	17	1.38	12.333	1.3622
−10	1	5.8	1.39	7.009	1.3635
−10	2	10	1.40	11.236	1.3854
0	0.1	2.5	0.40	2.821	0.3588
0	0.5	3.3	0.90	3.423	0.8365
0	1	3.8	1.10	4.396	1.1148
0	5	9.7	1.39	12.114	1.3674
10	1	3.0	0.71	3.363	0.8094
10	2	3.4	0.92	4.257	1.0870
20	1	2.7	0.40	2.979	0.5594

$$\left.\begin{aligned} G_1 &= \left(q_0 + p_1 q_1 \alpha_T^c \omega^c\right) / \left(1 + p_1^2 \alpha_T^c \omega^c\right) \\ \eta &= (q_1 - p_1 q_0)\alpha_T^d \omega^d / \left(q_0 + p_1 q_1 \alpha_T^{2d} \omega^{2d}\right) \end{aligned}\right\} \tag{5}$$

where q_0, q_1, p_1, c and d are parameters related to the properties of the VE dampers, which should be determined by statistical methods from the experimental data (Xu, 2001).

Equations (4) and (5) are the formulae of the equivalent standard solid model. In order to show the effects of temperature on the properties of VE dampers and verify the suitability of the equivalent standard solid model, the storage modulus G_1 and the loss factor η are calculated for different temperatures and frequencies using Equation (5). For the VE material 9050A, $q_0 = 2.7405 \times 10^6$, $q_1 = 3.3825 \times 10^5$, $p_1 = 0.0048$, $c = 1.34$, $d = 0.58$ (Xu et al., 2001). The calculated results and test results are listed in Table 1. Obviously, the storage modulus G_1 and the loss factor η of VE dampers vary sensitively with temperatures and frequencies. Analysis of the results shows that the numerical results of the equivalent standard solid model fit well with the experimental behaviour of VE dampers and the model can describe the influence of temperature and frequency on the properties of VE dampers.

3. TEST SETUP AND TEST PROGRAM

The test frame models are two identical 1/5-scale three-story plain reinforced concrete frames (i.e. the model scale factor $S_l = 0.2$). In order to increase the stability of the frame and to facilitate adding weights, two identical frames are used. A lumped mass system with a mass of 1354 kg for the first story, 1265 kg for the second story and 1130 kg for the roof is used to simulate the prototype structure. Overall, each test frame was 1.2 m in span, with story heights of 0.8 m for the first story and 0.66 m for each of the other two, as shown in Figure 2. The cross sections of the beams and columns are 50 × 110 mm and 80 × 80 mm, respectively.

The conventional VE damper, as shown in Figure 3, which consists of three steel plates clamping two VE material layers, is adopted. The VE material is 9050A material, made by Wuxi Shock Absorption Company in China. The shear area and the thickness of the VE layer are 60 × 50 mm and 5 mm, respectively.

Figure 2. The test frame model.

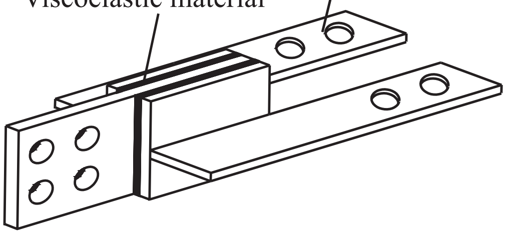

Figure 3. A conventional viscoelastic damper.

The test setup and instrumentation are designed to measure floor accelerations, lateral floor displacements, strains in beams and columns and the shear force of the VE dampers.

For this work, impact excitation and free-vibration tests were carried out to identify the dynamic characteristics of the structures before every major earthquake-simulation test. The time-scaled El Centro and Taft earthquake records are used as the seismic inputs for the shaking table tests. The peak accelerations are 0.12 g, 0.24 g, 0.50 g, 0.70 g, 1.00 g, 1.20 g and 1.40 g. Tests of the structure with and without VE dampers are carried out by installing or removing the bolts connecting the dampers and braces. It must be noted that tests of the structure without VE dampers were conducted only with the 0.12 g and 0.24 g El Centro records.

Table 2. Parameters of the test structure and dampers.

Parameters	floor	Mass (Kg)	Stiffness ($\times 10^6$N/m)	Story height (m)	Crack displacement (mm)	Crack shear force (KN)	Yield displacement (mm)	Yield shear force (KN)
The test structure	First	676.9	3.073	0.80	1.264	3.625	4.359	6.086
	Second	632.5	4.035	0.66	0.987	3.717	3.780	6.407
	Roof	565.0	4.035	0.66	0.842	3.717	3.440	5.802
VE dampers		VE layers			Shear area of each layer (m^2)		Thickness of each layer (mm)	
		2			3×10^3		5	

4. TEST RESULTS AND ANALYSIS

4.1. Comparison Between Test Results and Numerical Results

Two VE dampers were added to each floor of the test structures. Elastoplastic analysis of structures with VE dampers was performed, with the structure modeled using the trilinear stiffness degeneration model and the VE damper modeled using the equivalent standard solid model. The parameters of the test structure and the VE damper are given in Table 2. The parameters used in the equivalent standard solid model were $q_0 = 2.7405 \times 10^6$, $q_1 = 3.3825 \times 10^5$, $p_1 = 4.8 \times 10^{-4}$, $c = 1.34$, $d = 0.58$, and $T_0 = 94^\circ$C. The temperature during the tests was 8°C. The acceleration data measured by the accelerometer at the base plate was used as the numerical acceleration input for the earthquake wave.

The storage modulus G_1 and the loss factor η of the VE dampers were determined for the test temperature using the equivalent standard solid model. The equivalent stiffness k_d and the equivalent damping c_d can be calculated from the following formulae

$$k_d = n_v \sqrt{G_1^2 + G_2^2} A_v \cos^2 \theta / h_v \tag{6}$$

$$c_d = n_v G_2 A_v \cos^2 \theta / \omega h_v . \tag{7}$$

Where G_2 is the loss modulus of VE dampers, $G_2 = \eta G_1$, ω is the vibration frequency of the VE dampers (the first natural frequency of the structure with VE dampers will be adopted in the calculation), and θ is the angle in radians between the brace and the horizontal direction (in this test, $\theta = [0.186\ 0.161\ 0.161]\ \pi$). It should be noted from equations (6) and (7) that when the structure is damaged, the equivalent stiffness of the dampers k_d will not change, but the equivalent damping of dampers c_d will change with the natural frequency of the structure.

The stiffness matrix $\Delta\mathbf{K}$ and the damping matrix $\Delta\mathbf{C}$, which account for the contribution of the VE dampers to the structure, can be obtained through the following equations

$$\Delta\mathbf{K} = \begin{bmatrix} \Delta k_1 + \Delta k_2 & -\Delta k_2 & \\ -\Delta k_2 & \Delta k_2 + \Delta k_3 & -\Delta k_3 \\ & -\Delta k_3 & \Delta k_3 \end{bmatrix} \tag{8}$$

$$\Delta C = \begin{bmatrix} \Delta c_1 + \Delta c_2 & -\Delta c_2 & \\ -\Delta c_2 & \Delta c_2 + \Delta c_3 & -\Delta c_3 \\ & -\Delta c_3 & \Delta c_3 \end{bmatrix} \tag{9}$$

$$\left.\begin{aligned} \Delta k_i &= n_{di} \cdot k_d \\ \Delta c_i &= n_{di} \cdot c_d \end{aligned}\right\} \tag{10}$$

where Δk_i and Δc_i are the stiffness and the damping that the VE dampers in the i-th floor contribute to the i-th floor of structure, respectively, and n_{di} is the number of the VE dampers attached to the i-th floor of structure. The contributing stiffness matrix $\Delta\mathbf{K}$ and damping matrix $\Delta\mathbf{C}$ are added to the stiffness matrix and the damping matrix (respectively) of the structure with VE dampers, and the elastoplastic dynamic responses of the structure with VE dampers can then be analyzed.

Figure 4 shows the experimentally obtained and numerically simulated time-history response to the 0.12 g El Centro earthquake of the structure with two VE dampers for each floor. Figures 5 and 6 show the same results, but for the 0.12 g Taft earthquake and the 0.24 g El Centro earthquake, respectively. During the tests, cracks can be found in the structure when the earthquake wave reaches 0.24 g. This shows the experimental structure has entered its nonlinear range. Under the 0.24 g El Centro earthquake wave, the maximum experimental and numerical displacement responses of the roof are 4.002 mm and 3.468 mm, respectively, and the maximum experimental and numerical acceleration responses of the roof are 3.23 m/s^2 and 3.05 m/s^2, respectively. It can be seen from Figures 4 to 6 that the numerical results agree well with the experimental data for both earthquake waves, which indicates that the numerical results of the structure with VE dampers are adequately accurate when the trilinear stiffness degeneration model is used to simulate the structure and the equivalent standard solid model is used to simulate VE dampers. The error between the numerical and experimental results may be caused by one or more of the following: (1) the calculated stiffness and mass of the structure differ from the actual ones; (2) the calculated stiffness of the VE dampers is different from with the real one; (3) the calculated damping of the structure differs from the measured values; (4) the simplifications implicit in the elastoplastic analysis of the structure.

4.2. Comparison between the structure with dampers and without dampers

In order to study the shock absorption ability of VE dampers, the dynamic responses of the structure with and without VE dampers are plotted together in Figure 7, which shows the experimentally obtained dynamic response of the structure with two dampers on each floor and the structure without dampers under the 0.24 g El Centro earthquake.

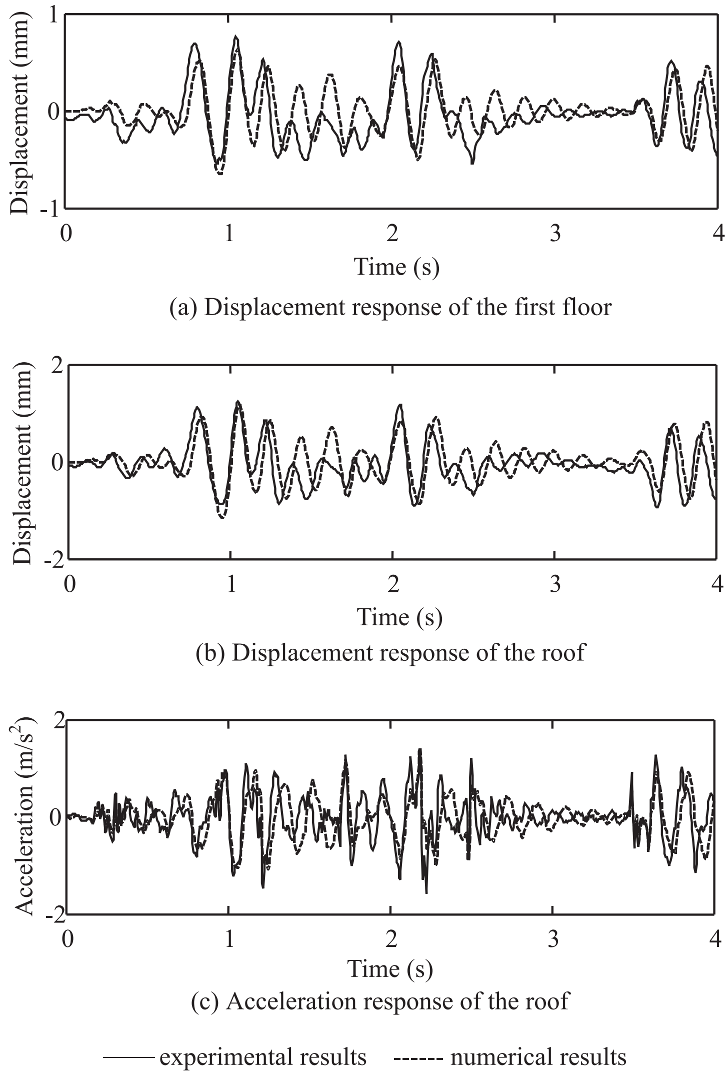

Figure 4. Comparison between the experimental and numerical results for the viscoelastic structure under the 0.12 g El Centro earthquake.

It can be clearly seen from Figure 7 that the displacement and acceleration responses of the structure with VE dampers are smaller than those of the structure without VE dampers. It can also be seen that the reduction in the acceleration is smaller than the reduction in the displacement. The experimental maximum lateral roof displacement of the structure without dampers is 7.284 mm, while the experimental maximum lateral roof displacement of the structure with dampers is 4.002 mm; a reduction of 45.1%. The experimental maximum roof acceleration of the structure without dampers is 5.10 m/s^2, while the experimental maximum roof acceleration of the structure with dampers is 3.23 m/s^2, a reduction of 36.7%. These indicate that when VE dampers are installed in the structure, the damping and the stiffness

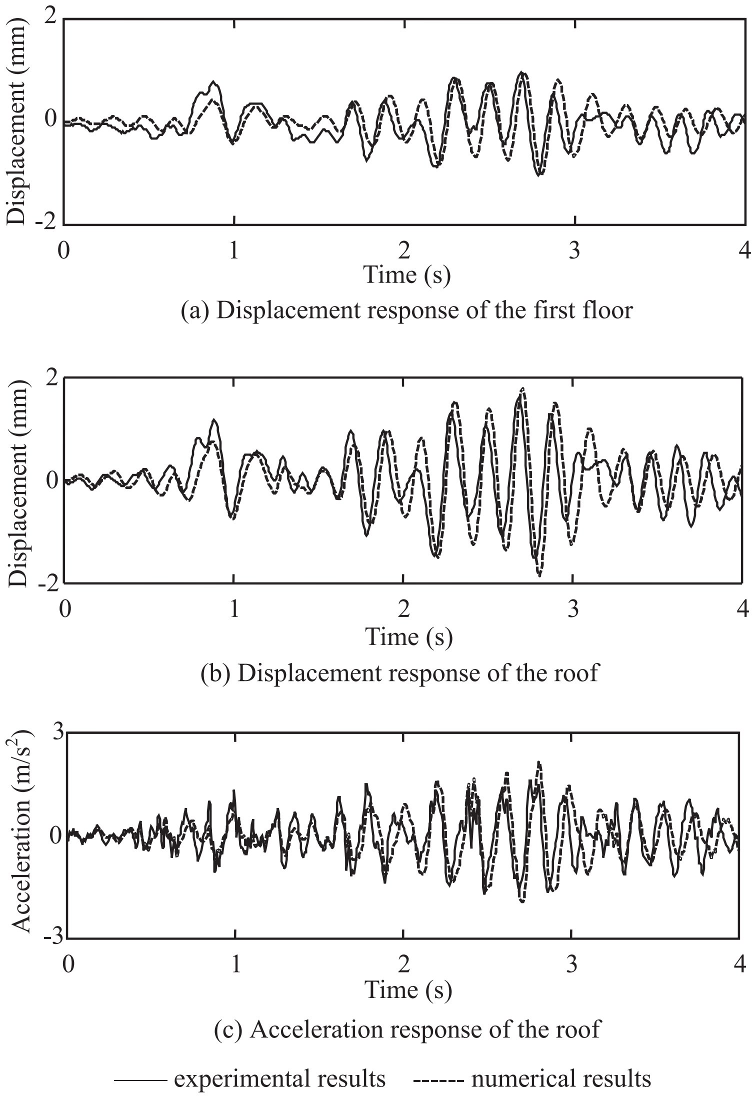

Figure 5. Comparison between the experimental and numerical results of the viscoelastic structure under the 0.12 g Taft earthquake.

of the structure are increased and the dynamic responses of the structure are reduced. In this test, the displacement response was reduced by about 40–55%, and the acceleration response was reduced about 30–40%. Experimental study shows that the reductions of displacement and acceleration are more evident for stronger earthquakes, because the interstory drifts of the structure increase with increasing earthquake excitation amplitude, and thus the shear deformations and the energy dissipation of the VE dampers are also increased, and the dynamic responses are reduced more effectively.

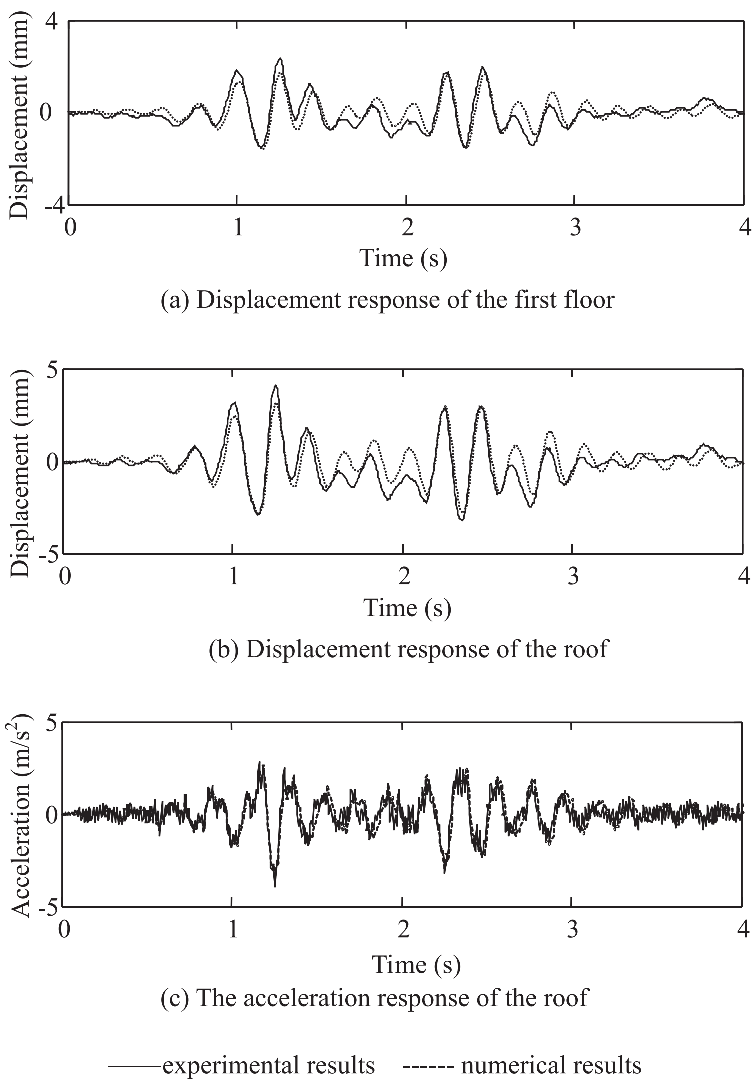

Figure 6. Comparison between the experimental and numerical results of the viscoelastic structure under the 0.24.g El Centro earthquake.

4.3. Frequency Field Analysis

In order to show the earthquake mitigation effects of VE dampers more clearly, frequency field analysis on structures with and without VE dampers were conducted. Figure 8 shows the roof acceleration response energy spectra for both numerical and experimental structures with two VE dampers in each floor and the experimental structure without VE dampers, all responding to the 0.24 g El Centro earthquake wave. When VE dampers are added to the structure, the peak acceleration response energy index is reduced from 576 to 290. At the same time, the peak acceleration response energy index moves to the right, showing that

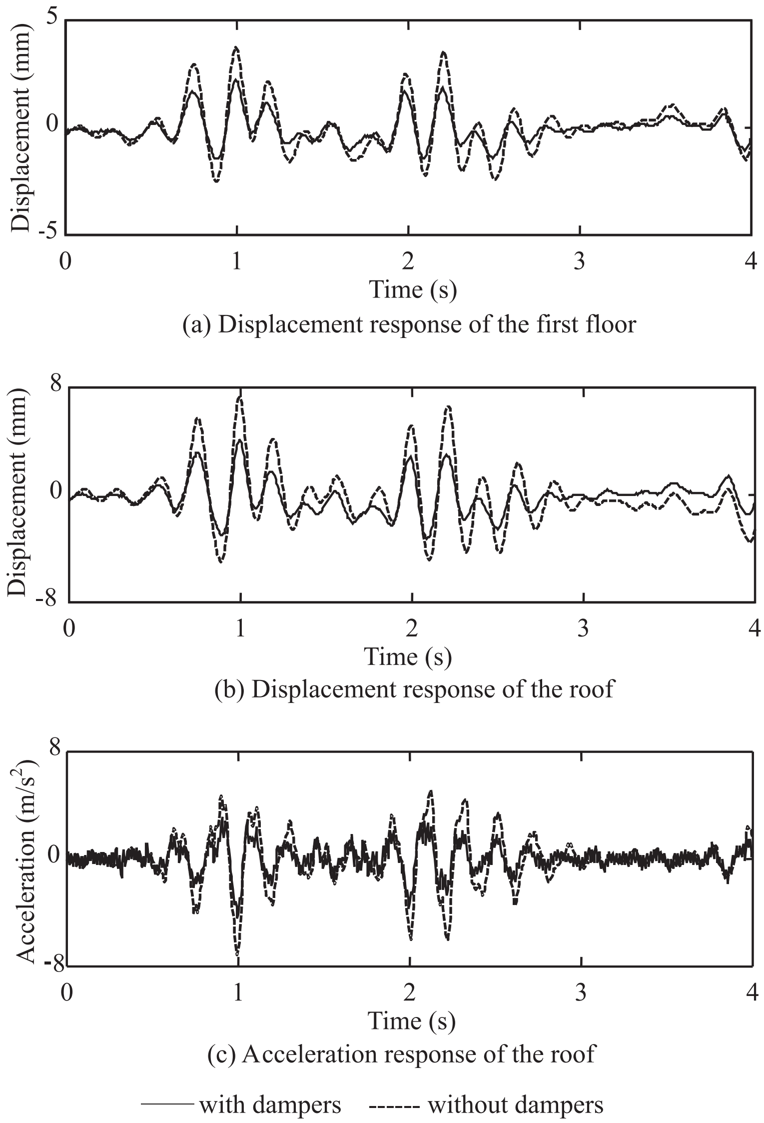

Figure 7. Comparison between the dynamic responses of structures with and without dampers.

the first natural frequency (corresponding to the peak acceleration response energy index) is increased: The experimental frequency of the structure without dampers is 5.91HZ, the experimental frequency of the structure with dampers is 6.42HZ, and the calculated frequency for the structure with dampers is 6.10HZ. This increase in the natural frequency is due to the increase in stiffness produced by the dampers. It can also be seen that the numerical results fit well with the experimental results, verifying the accuracy of the adopted model. It must be noted the natural frequencies here are derived from the roof acceleration frequency spectra analysis, and are slightly different from the following natural experimental frequencies

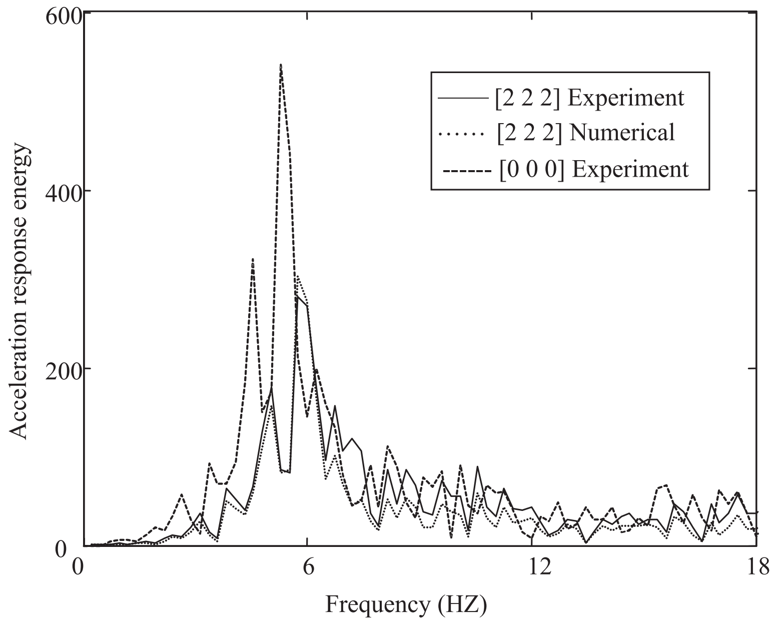

Figure 8. Comparison of frequency field structure response spectra.

(derived from the inherent vibration analysis software of shaking table tests) and numerical frequencies (derived by solving the eigenequations the of structures), although these natural frequencies are very near to one another for the same structure.

4.4. Dynamic Characteristics

The j-th modal equivalent damping ratio can be determined by the modal strain energy method (Ou and Zou, 1999; Xu, 2001).

$$\zeta_j = \frac{E_{dj}}{4\pi E_j} = \frac{\eta \Phi_j^T \Delta \mathbf{K} \Phi_j}{\Phi_j^T \mathbf{K} \Phi_j} \tag{11}$$

where E_{dj} is the per-cycle energy dissipation of the VE dampers for the j-th modal shape, E_j is the j-th modal strain energy of the structure, η is the loss factor of VE dampers, $\Delta\mathbf{K}$ is the stiffness matrix due to the contribution of the VE dampers, $\mathbf{K}$ is the stiffness matrix of the structure with VE dampers, and Φ_j is the j-th modal shape of the structure with VE dampers.

The natural frequency f and the damping ratio ζ of the structure with VE dampers can be calculated once the stiffness matrix and the damping matrix of the structure with VE dampers have been determined. The experimental frequency f and damping ratio ζ are determined using the coupled vibration analysis software from the shaking table tests in accordance with the results of impact tests. For the test frame without VE dampers, the experimental first modal natural frequency f is 6.01HZ and the experimental first modal damping ratio ζ is

Figure 9. Damage characteristics of the model frame.

5.12%, while the analytical first modal natural frequency f is 5.35HZ, and the first modal damping ratio ζ of reinforced concrete structure is 5.00% in accordance with criterion of reinforced concrete structure. For the test frame with VE dampers, the experimental first modal natural frequency f is 6.97HZ and the experimental first modal damping ratio ζ is 7.01%, while the analytical first modal natural frequency f is 6.02HZ and the first modal damping ratio ζ is 6.82%. Experimental results show that when VE dampers are added to the structure, the first natural frequency f and the first damping ratio ζ of the structure increase by 15.97% and 36.92%, respectively. It was also shown that the numerical results agree well with the experimental results. The experimental results are slightly larger than the numerical results because the calculations of the stiffness and the damping of the structure are not accurate enough.

4.5. Damage Situation

The appearance of cracks was monitored during the experiment. Cracks were first located in the middle part of the beams, under the 0.24 g El Centro earthquake. They subsequently subsequently appeared at the ends of the beams, and at the top and bottom ends of the columns. According to previous experience of shaking table tests, the bottom ends of the columns in the first floor were the first components to crack and yield, these were given additional steel reinforcements to ensure the success of the experiment. In such cases, the cracks transfer from the bottom of the first floor columns to the columns' central part, as shown in Figure 9. When the structure was subjected to the 1.4 g El Centro earthquake, the base slab slid and was destroyed, the added weight in the roof jumped, and plastic hinges formed in the columns of the first floor.

Finally, under a 0.5 g resonant frequency (5.9HZ) sine wave, the structure was further damaged. A 12 mm width crack appeared, the steel began to yield in a beam in the roof floor (caused by the roof weight's bump), and 3.5 mm width cracks formed in the columns of the first floor. Under the 0.24 g earthquake (medium earthquake) cracks began to arise in the structure, primarily at the bottom of columns, and at both ends and the mid-span of beams. Even under the 0.7 g earthquake (strong earthquake) the structure was not seriously damaged and could be used after repair, demonstrating that the structure with VE dampers has excellent seismic resistance. After the 1.4 g El Centro earthquake and the 0.5 g resonant sine wave, the VE dampers themselves were still not destroyed. This indicates that VE dampers have excellent limit deformation ability.

5. CONCLUSIONS

This paper summarizes an experimental and numerical study of VE dampers as energy dissipation devices in structural applications under seismic loads. The following conclusions can be obtained through the experimental and numerical study.

(1) When VE dampers are added to the structure, the stiffness and the damping of the structure were increased, thus increasing the natural frequency and the damping ratio of the structure.
(2) The numerical method can accurately predict responses of the structure with VE dampers when the trilinear stiffness degeneration model is used to simulate the structure's behavior and the equivalent standard solid model is used to simulate the VE dampers.
(3) VE dampers are effective in reducing the dynamic responses of the structure under earthquake ground motions. In general, in the test described, the displacement responses can be reduced by 40–55%, and the acceleration responses by 30–40%.
(4) VE dampers have excellent energy absorption ability and limited deformation ability.

Acknowledgement. Financial support for this research was provided by the National Natural Science Foundation of China (grant number 50508010), Jiangsu Province Natural Science Foundation in P.R.China (grant number: BK2005410), the Teaching and Research Award Program for Outstanding Young Teachers of Southeast University in P.R.China. This support is gratefully acknowledged.

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