Scaling Law in Spatial Networks

——Its Effects on Topology and Dynamics

Zengru Di

Beijing Normal University, China

Collaborators

- Department of Systems Science, BNU, China
  - Yanqing Hu
  - Hua Yang
  - Luo Dan
  - Yougui Wang
  - Ying Fan
  - Zengru Di
- Department of Physics, BIU, Israel
  - Daqing Li
  - Shlomo Havilin

Outlines

- Motivations
- Discovering What
- Understanding Why
- Investigating Its Impacts
  - Synchronization
  - Traffic Dynamics
  - Epidemic Process
- Conclusions

Systems Science

To Provide a Systematic (Unified, Universal) View for Varies Complex Systems by a Systematic Approach.

Complex System

A complex system composed of many interactive agents, can have qualities not directly traceable to the system's components, but rather to how those components interact. These new qualities are irreducible to the system's constituent parts . The whole is greater than the sum of its parts.

北京师范大学系统科学系

- 复杂系统基本理论
  - Complex Networks
  - Nonlinear Dynamics, Pattern Formation
- 社会经济系统分析
  - Financial Market
  - Econophysics, Money Circulation
  - Human Resource Management
- 生命与生态复杂系统
  - Neuro-Networks
  - Cognition and Learning
- 多主体系统与演化算法
  - Complex Adaptive System
  - Genetic Algorithm

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复杂系统基本理论

非线性系统时空结构、复杂网络的结构与动力学、复杂系统的演化特征和临界行为等基本性质和规律。

社会经济系统分析

将经济看作一个演化的复杂系统，分析实际经济问题，探索经济系统的核心规律，为决策提供理论和实证支持。

生命与生态复杂系统

生命、生态、神经系统的理论研究，神经元动力学、高级认知过程如学习、工作记忆、决策的动力学机制。

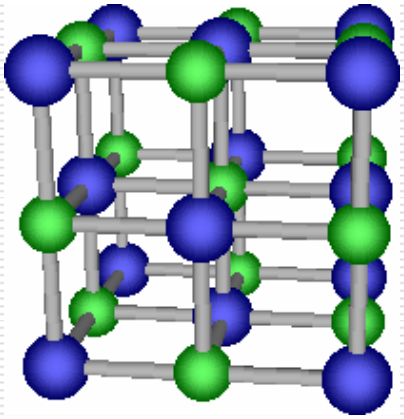
多主体系统与演化算法

微观个体相互作用与宏观集体行为的对应机制，个体间的协作机制与计算智能。

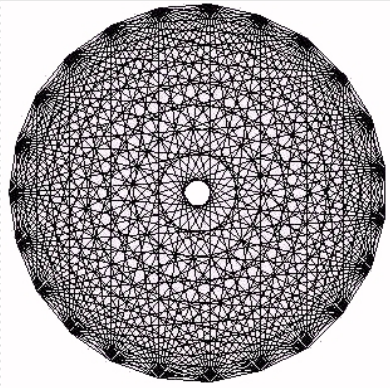
Reductionism and complexity

Connection topology

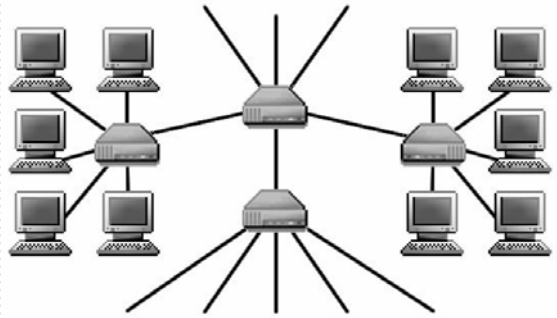
Crystal Lattices

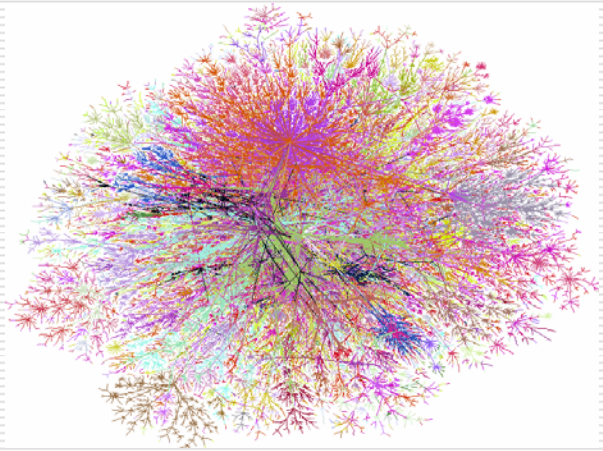


All-to-all interactions



Internet





Diffusion

Mean field

Complex Networks

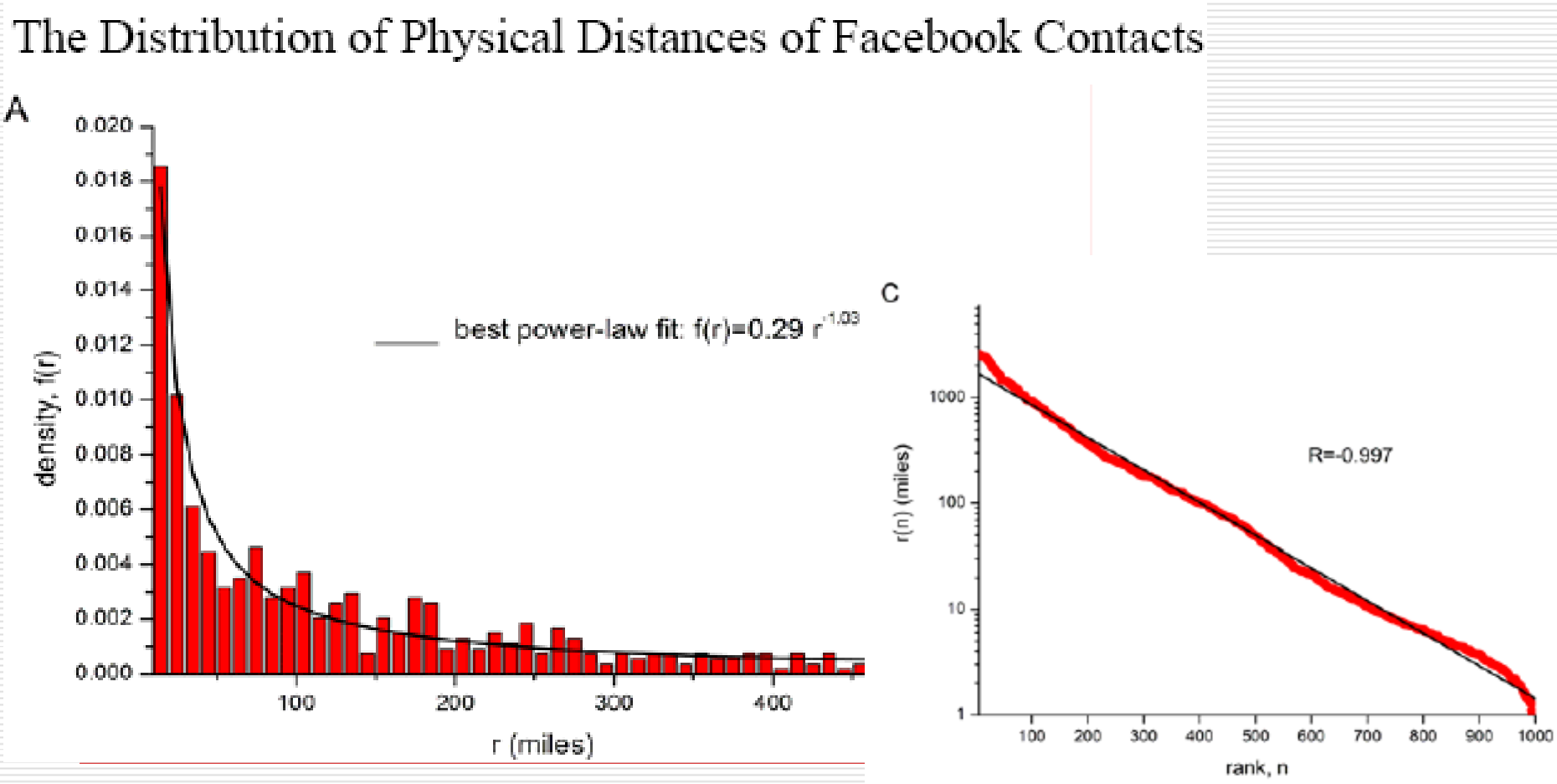
complex networks are the backbone of complex systems

- every complex system is a network of interaction among numerous smaller elements
- understanding a complex system = break down into parts + reassemble
  - network anatomy is important to characterize because structure affects function (and vice-versa)
  - ex: structure of social networks
    - prevent spread of diseases
    - control spread of information (marketing, fads, rumors, etc.)
  - ex: structure of power grid / Internet
    - understand robustness and stability of power / data transmission

Spatial Properties of Complex Networks

Distance Is Not Dead:

Social Interaction and Geographical Distance in the Internet era



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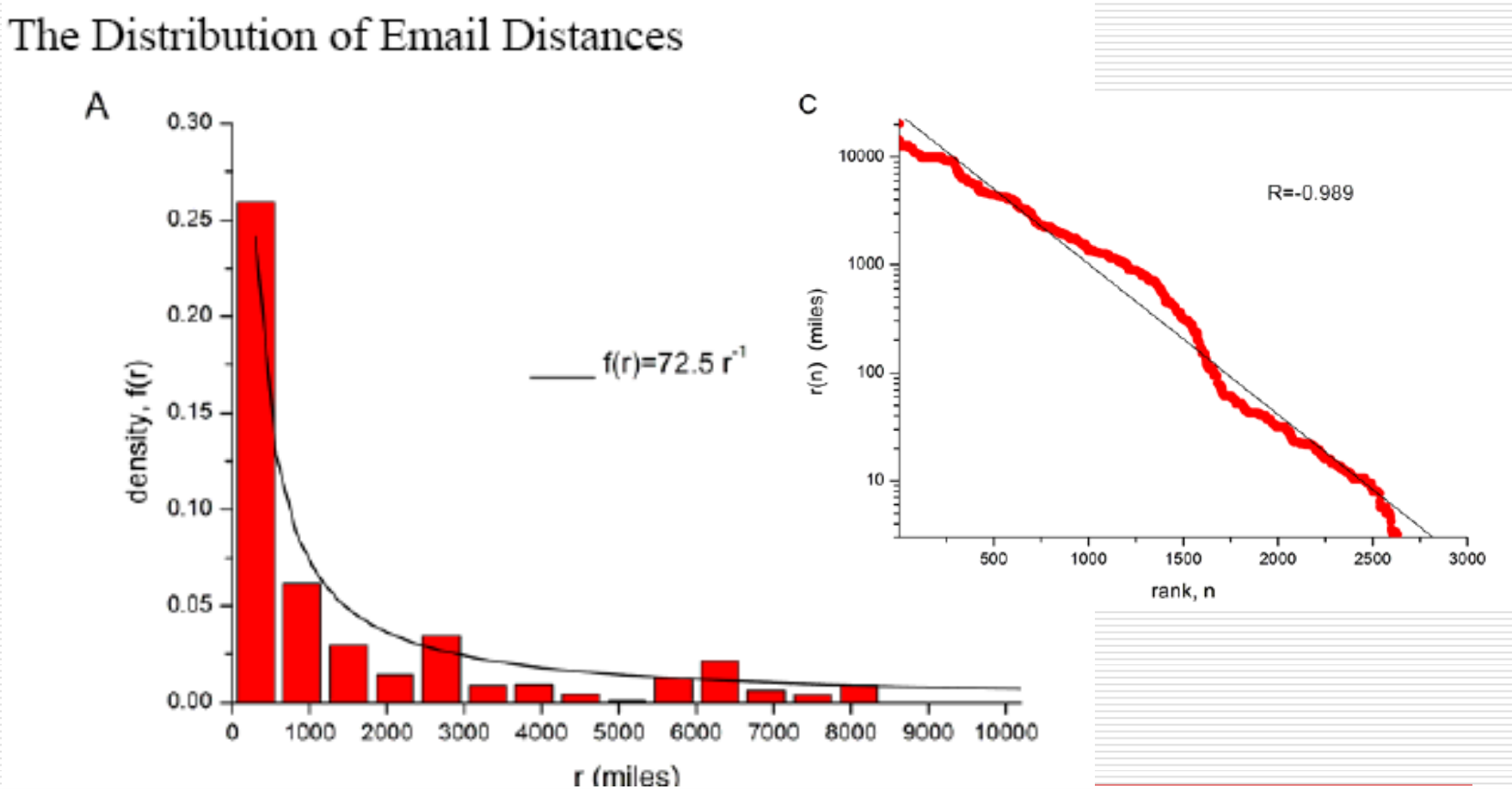
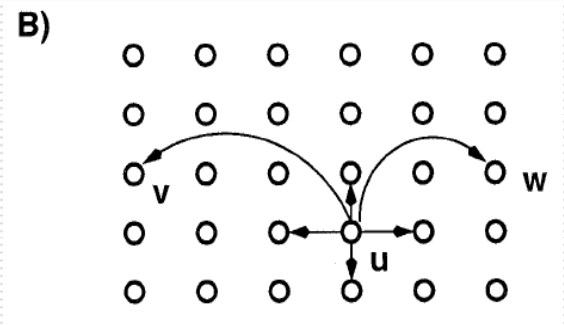


Discovering the Spatial Structures Of Networks

Social Networks

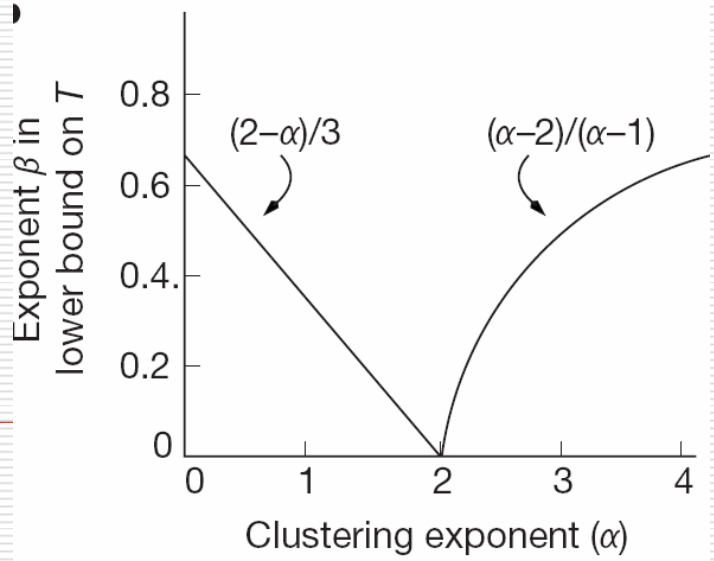
Milgram: Six degrees of separation.

Travers, J. and Milgram, S., *Sociometry*. 32, 425-443 (1969).

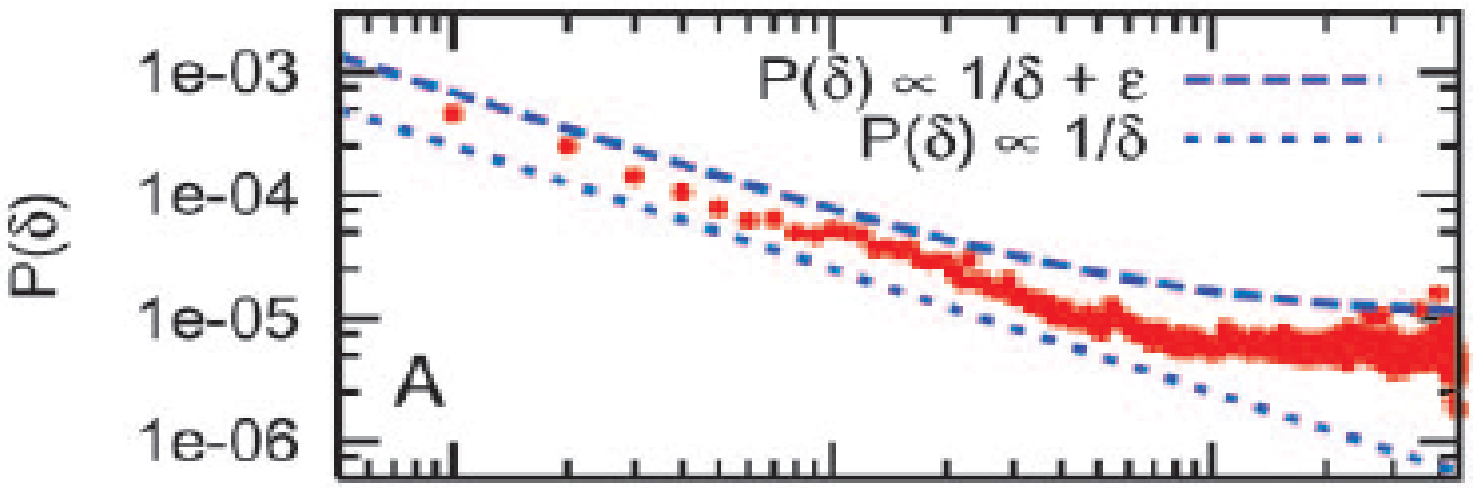


Kleinberg:

Kleinberg, J. M. Navigation in a small world. *Nature*. 406, 845 (2000)



Empirical Results



D.Liben-Nowell, J. Novak, R. Kumar, P. Raghavan, and A. Tomkins. PNAS 102, 11623-11628, (2005).

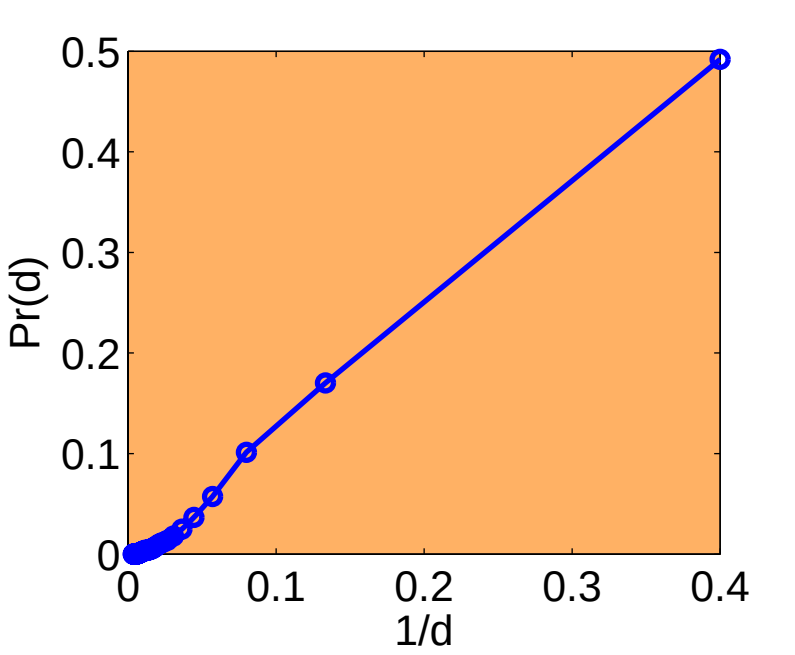
Empirical results

R. Lambiotte and his cooperators: 2.5 million mobile phone customers

Physica A. 387(2008)

The distribution of geographic distance between friends is

P(d)~ d^-1



Transportation Networks

Y. Hayashi, A review of Recent Studies of Geographical Scale-Free Networks, IPSJ Digital Courier, Vol.2, 155-164 (2006).

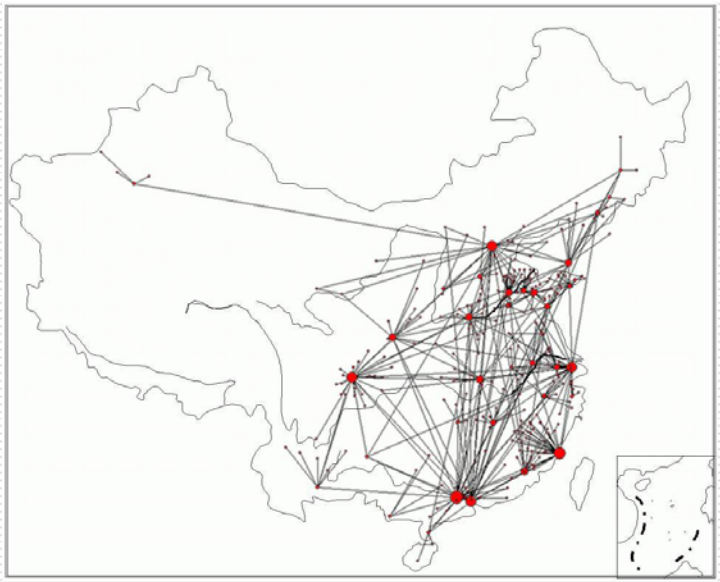
Japan: An airline with international line shows power law distance distribution

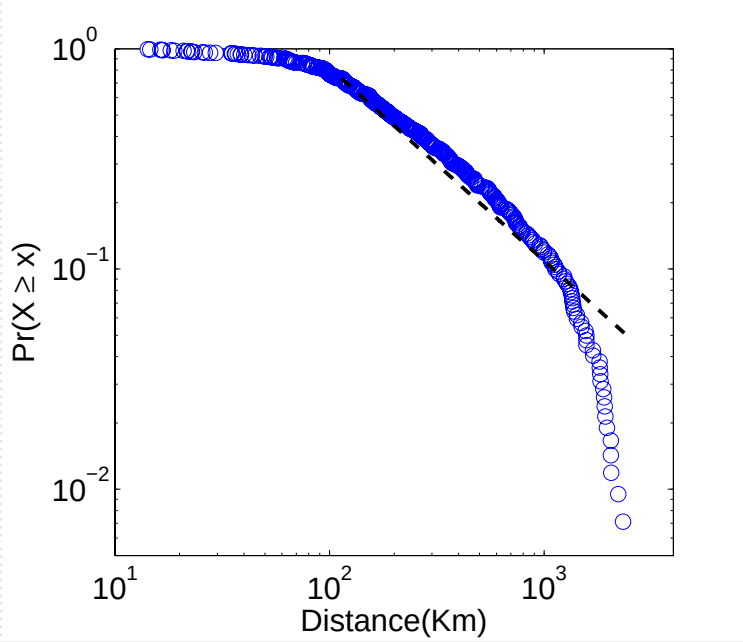
Zengwang Xu, Robert Harriss, Exploring the structure of the U.S. intercity passenger air transportation network: a weighted complex network approach, GeoJournal 73:87102,(2008).

U.S.: Domestic airline has fat tail with -2.2

Transportation Networks

The Express Transport Network





- **The accumulative distribution of distance in the ETN A (−1.87).**

Understanding Why Do the Networks Possess Spatial Scaling Properties

Getting information through social network

$$Max\ \varepsilon = -\sum_{i=1}^{n} q_i \ln q_i$$

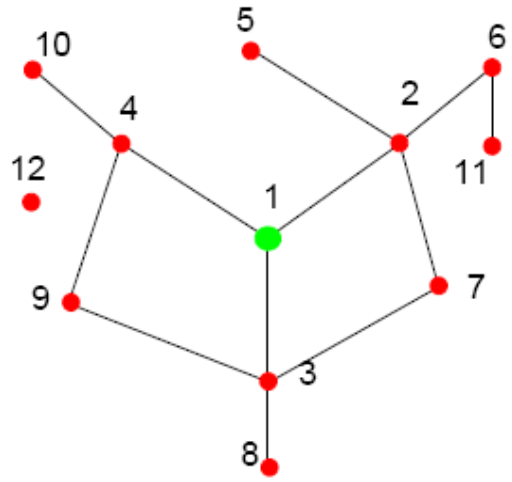


FIG. 1: The friends of node 1. Node 2, 3 and 4 are the friends of node 1 which Eq. (2) yields that $d(1,2)+d(1,3)+d(1,4) = w$. The size of the network is $n = 12$ and the information sequence is $\{2,3,4,5,6,7,7,8,9,9,10\}$ and the frequencies of all nodes are $q_2 = q_3 = q_4 = q_5 = q_6 = q_8 = q_{10} = \frac{1}{11}$, $q_7 = q_9 = \frac{2}{11}, q_1 = q_{11} = q_{12} = 0$.

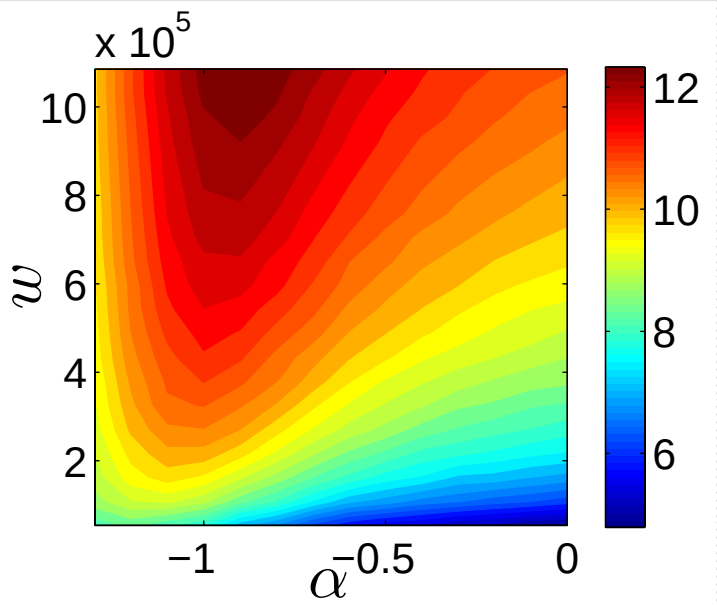
Understanding Why Do the Networks Possess Spatial Scaling Properties

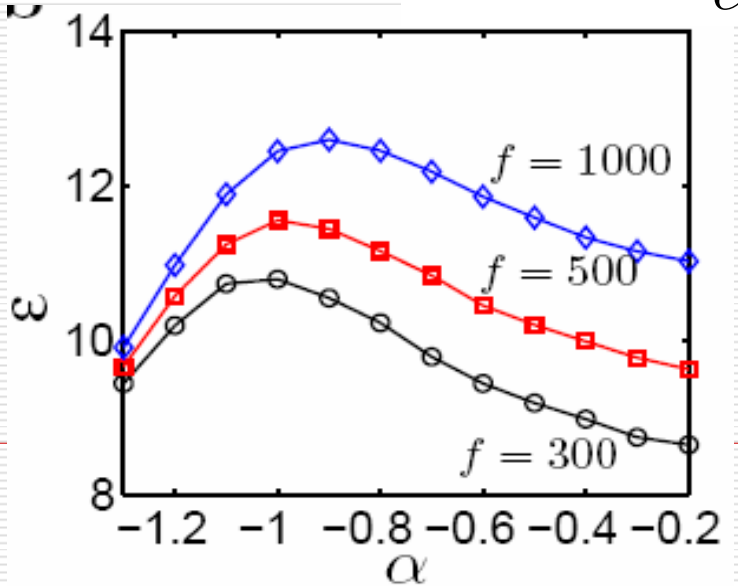
Maximization of Entropy

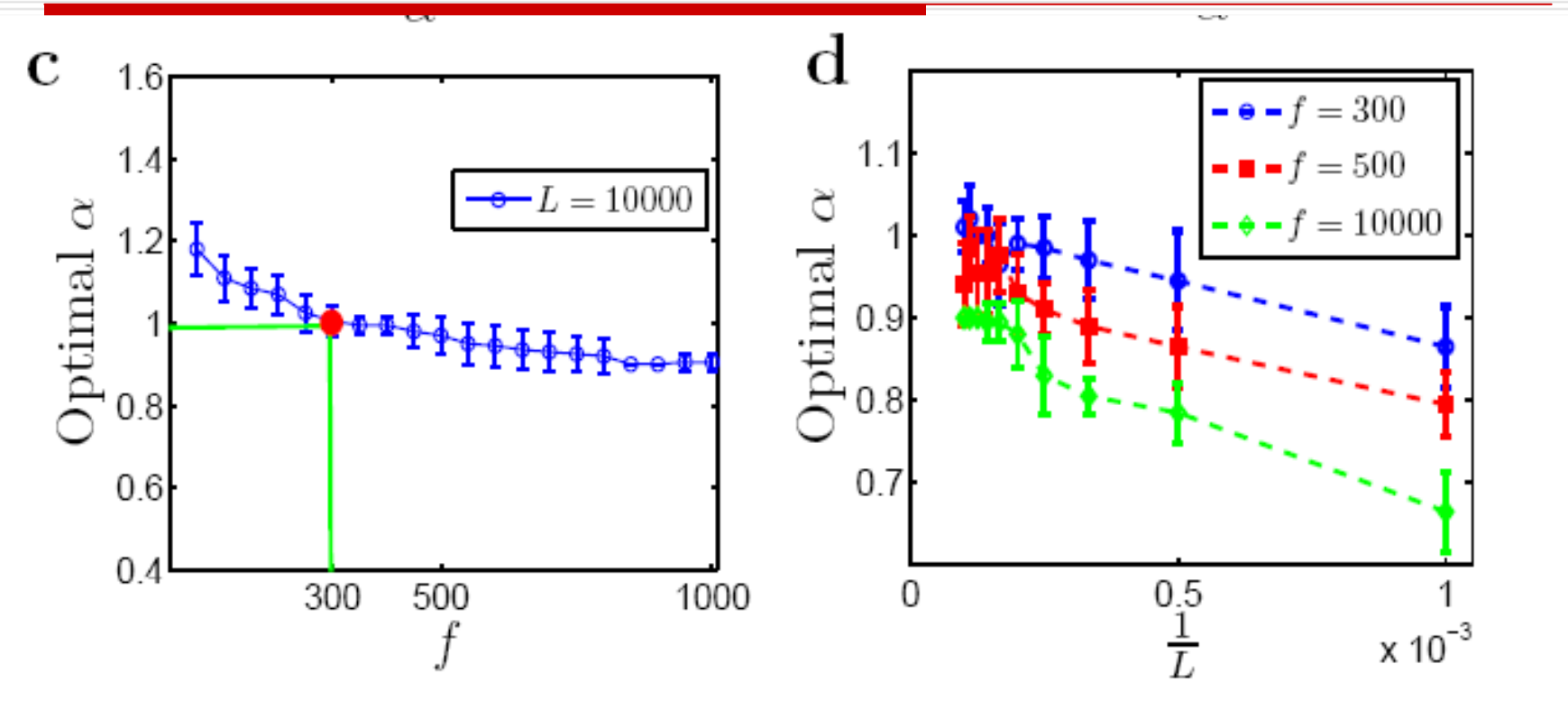
$$\max \quad \varepsilon = -\sum_{i=1}^{n} p_i \ln(p_i)$$

$$st. \begin{cases} \sum_{j=1}^{m} d(1, j) = w \\ p(d) \sim d^{\alpha} \end{cases}$$

α ?







$$P(r) \propto r^{-\alpha}$$