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Accelerating consensus of self-driven swarm via a weighted model

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HIGHLIGHTS

- A weighted self-driven swarm model is proposed.
- The impact of the neighbor on the center agent is determined on the view angle between them.
- There exists an optimal phenomenon leading to the shortest convergence time.
- The optimal phenomenon is robust to the density, the absolute velocity of swarms, and the strength of noise.

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ABSTRACT

In this paper, we study a weighted self-propelled agent system, wherein each agent's direction is affected by its spatial neighbors with different impacts. In the model, a tunable parameter $\alpha \geq 0$ is introduced to weight the different impacts of spatial neighbors: if $\alpha = 0$, the agent's direction is updated by averaging all of neighbors directions and own direction, i.e., Vicsek model. Otherwise, with the increase of the value of α, the agent's direction is more affected by the agent who has small view angle between them. Interestingly, simulation results show that there exists an optimal α leading to the shortest convergence time. Thus, our findings provide a powerful mechanism for collective motions in biological and technological multiagent systems.

1. Introduction

The collective motion is a ubiquitous phenomenon in the nature, ranging from the behavior of groups of ants, colonies of bacteria and clusters of cells in the microcosmic scale, to migration of flocks of birds and schools of fish in the macroscopical scale [1–9]. To understand the basic mechanism leading to the consensus of collective population, one prototypical model was proposed by Vicsek in 1995 [10] (labeled the Vicsek model). In the Vicsek model, N self-propelled agents are driven toward different directions with a constant absolute velocity in a squared zone with periodic boundary condition. Meanwhile, each agent synchronously updates its direction by averaging all directions of the agents within the horizon radius R of the agent (including the agent itself). The simple model shows that when the density of the system is high and the noise is small enough, all agents will converge to the same direction.

In recent years, due to simplicity and efficiency, many modified versions of the Vicsek model were proposed. For example, the models with adaptive velocities were proposed in Refs. [11–13]; the model with heterogeneous radii was introduced in

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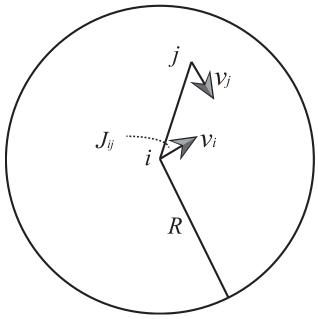


Fig. 1. The diagram of the model. J_{ij} denotes the view angle between agent i and its neighbor j.

Ref. [14]; a model with avoidance of collisions was studied in Ref. [15]; models with limited view angles were investigated in Refs. [16–20]. In addition, the models with effective leaderships were introduced in Refs. [21–24]. Most previous works, however, assumed that each agent updates its direction by averaging all agents' directions within certain scope. In reality, on one hand, most animals are incapable of seeing the neighbors following them. For example, the cyclopean retinal field of human is about 180° and the cyclopean retinal field of tawny owl is 201° [17,19]. On the other hand, when animals and persons are moving, the impacts of neighbors in front of the agent are often larger than the impacts of neighbors at side of the neighbors behind. Thus, to mimic such common phenomenon in the nature, in this paper, we propose a weighted self-propelled agent system, where the impact of a neighbor on the center agent depends on the view angle between them. The smaller view angle between them is, the larger impact of the neighbor on the center agent is. Extensive numerical simulations show that there exists an optimal phenomenon leading to the agents' direction consensus fastest. In addition, we find that such phenomenon is robust to the density, the absolute velocity of swarms, and the strength of noise.

The paper is organized as follows. The model is described in Section 2. The simulation results and qualitative analysis are given in Section 3. The work is concluded in the last section.

2. Model

In the original Vicsek model, the position of agent i is updated according to Refs. [10,25]

$$x_i(t+1) = x_i(t) + v\mathrm{e}^{i\theta_i(t)}. \tag{1}$$

Here v is the absolute velocity, $\theta_i(t)$ is the direction of the movement. In addition, at time step t, each agent averages all the agents' directions within the horizon radius R of the agent (including itself) as its direction of time $t+1$, so its direction is updated as

$$\mathrm{e}^{i\theta_i(t+1)} = \mathrm{e}^{i\Delta\theta_i(t)} \frac{\sum\limits_{j\in\Gamma_i(t)} \mathrm{e}^{i\theta_j(t)}}{\left\| \sum\limits_{j\in\Gamma_i(t)} \mathrm{e}^{i\theta_j(t)} \right\|_2}. \tag{2}$$

Where $\Delta\theta_i$ denotes the noise of the system, and $\Gamma_i(t)$ is the set of neighbors for agent i at time step t (including itself).

From Eq. (2), we can find that, in the Vicsek model, the impacts of neighbors on the center agent are the same regardless of their locations or directions. As we mentioned, individuals may be easily affected by the neighbors in front of them. So, to reflect such fact, we first define the view angle between the agent i and one of its neighbor j, J_{ij}, which is the angle between the line individual j to i and velocity of individual i. The sketch of view angle is illustrated in Fig. 1.

The impact of the agent j on the center agent is given as the function of the view angle J_{ij}

$$W_{ji} = \frac{\mathrm{e}^{-\alpha J_{ij}}}{\sum\limits_{j\in\Gamma_i(t)} \mathrm{e}^{-\alpha J_{ij}}}. \tag{3}$$

α is a tunable parameter to adjust the different impacts of neighbors' directions on the center agent. The term $\sum_{j\in\Gamma_i(t)} \mathrm{e}^{-\alpha J_{ij}}$ in the denominator of Eq. (3) is to normalize the impacts of different neighbors agents and itself. In this paper, we do not consider the case of $\alpha < 0$, because it means that agents are easily affected by the neighbors behind them, so it is somewhat irrational. $\alpha = 0$ means that the impacts of neighbors are equally considered, i.e., the Vicsek model. When $\alpha > 0$, the smaller view angle J_{ij} between agent i and neighbor j is, the larger impact of j on the next direction of agent i is. Namely,

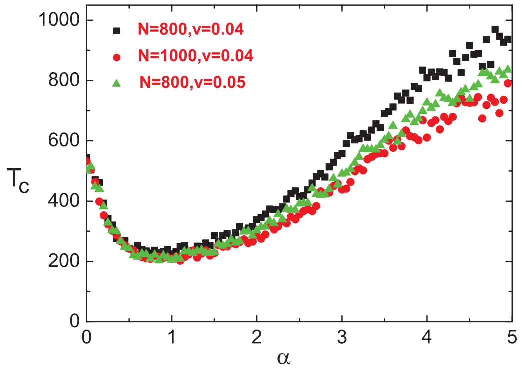


Fig. 2. The relationship between the transient time step T_c and the parameter α for different number of agents N and different absolute velocities v. All the data points above are obtained by averaging over 500 different realizations. Here $R = 0.5$.

neighbors in front have greater impacts on the center agent. Now, in our weighted model, the direction of agent *i* is updated as

$$e^{i\theta_i(t+1)} = e^{i\Delta\theta_i(t)} \frac{\sum_{j\in\Gamma_i(t)} W_{ji} e^{i\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t)} W_{ji} e^{i\theta_j(t)} \right\|_2}. \tag{4}$$

3. Results and discussion

In order to measure the degree of consensus for all the agents, an order parameter is introduced as

$$V_\alpha = \frac{1}{N} \left\| \sum_{i=1}^{N} e^{i\theta_i(t)} \right\|_2 . \tag{5}$$

A larger value of V_α indicates a better consistent system, especially when $V_\alpha = 1.0$, all agents move in the same direction.

In the beginning, N agents are randomly distributed to a square of size 10×10 with periodic boundary conditions. To quantify the speed of consensus of the self-driven system, we study the transient time step T_c, which is defined as the time step when the order parameter V_α first surpasses a threshold value 0.995. We have checked that qualitative results are not changed when the threshold values are large enough.

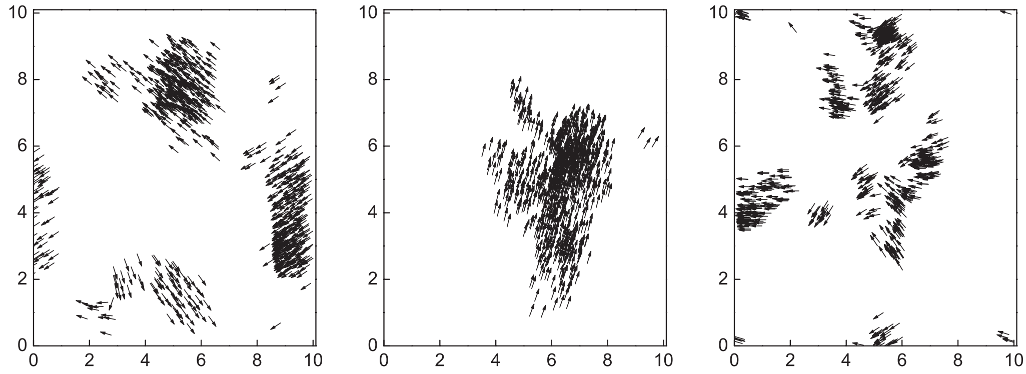
Fig. 2 displays the transient time step T_c in dependence on the value of α with different values of N and different absolute velocities v. As shown in Fig. 2, one can find that there exists an optimal value of $\alpha_{opt} \cong 1.0$ leading to the shortest T_c. Here we give a qualitative explanation to such a non-monotonic phenomenon. As we all know, when the value of α is sufficiently small, the model is similar to the Vicsek model. In this case, as stated by Ref. [19]—the existence of superfluous communications in the Vicsek model, which may counteract the direction consensus. As a result, small α is not conducive to playing the roles of neighbors in front of the center agent and leading to longer convergence time. On the contrary, for large value of α, the direction of the center agent is only affected by *few* neighbors ahead and own direction, in this case, each agent almost does not change its direction, which is also against the consensus of self-driven system. Thus, moderate α not only incorporates the greater impacts of neighbors in front but also encloses appropriate number of neighbors to follow the main direction of the system, leading to the most efficiency of consensus.

To intuitively display the effects of the value of α on the transient time step T_c, Fig. 3 illustrates the locations and velocities of all the agents at the 200th time step with $N = 800$ and $v = 0.04$. One can find that, when $\alpha = 1$, almost all of agents are moving in the same direction, however, agents are still moving in different directions when $\alpha = 0$ or $\alpha = 3$. The results in Fig. 3 again verifies that there exists an optimal value of α leading to the most efficiency of consensus.

Whether the optimal phenomenon also exists in other situations? To verify our conjecture, we change the number of agents N and the absolute velocity v (even when the v is large or small). Interestingly, as shown in Figs. 4 and 5, the optimal phenomenon is robust to the number of agents N and the absolute velocity v (in other word, the result is also robust to the density of agents, since the density of agents changes with the number of agents when the size of square is unchanged). What's more, the optimal value of α_{opt} is still about 1.0.

When compared with the *optimal view angle model* in Ref. [19], the advantages of our model can be summarized as follows:

(A) In our model, neighbors in different locations have different impacts on the center agent. What's more, the impacts of the neighbors behind are also considered. Yet, in optimal view angle model, only the impacts of neighbors within the given region are considered and the impacts of the neighbors are the same.



(a) $\alpha = 0$. (b) $\alpha = 1$. (c) $\alpha = 3$.

Fig. 3. Illustrations of locations and velocities of agents at 200th time step with (a) $\alpha = 0$; (b) $\alpha = 1$ and (c) $\alpha = 3$ respectively. Here $R = 0.5$, $N = 800$ and $v = 0.04$.

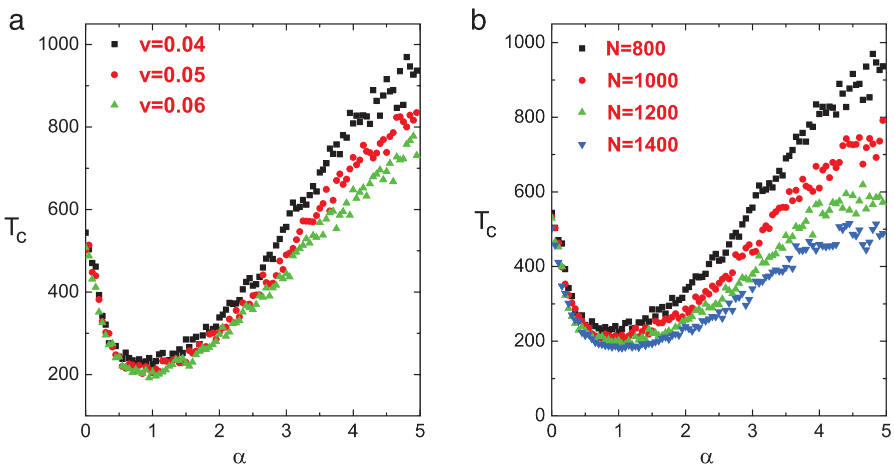


Fig. 4. (a) For different values of v, the effect of α on the transient time step T_c with fixed $N = 800$; (b) For different values of N, the effect of α on the transient time step T_c with $v = 0.04$. All the data points above are obtained by averaging over 500 different realizations. Here $R = 0.5$.

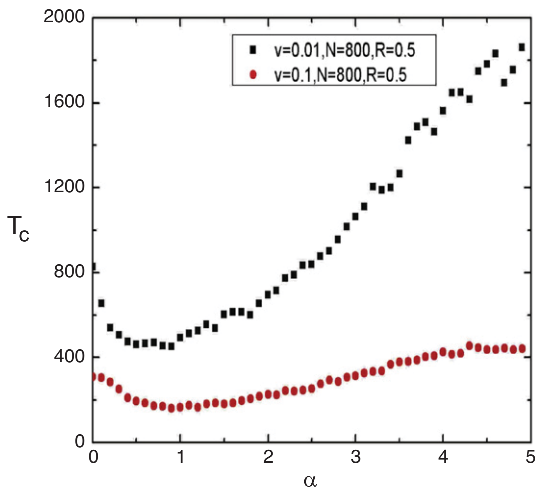


Fig. 5. α_{opt} is still about 1.0 when v is large or small. All the data points above are obtained by averaging over 500 different realizations. Here $R = 0.5$, $N = 800$.

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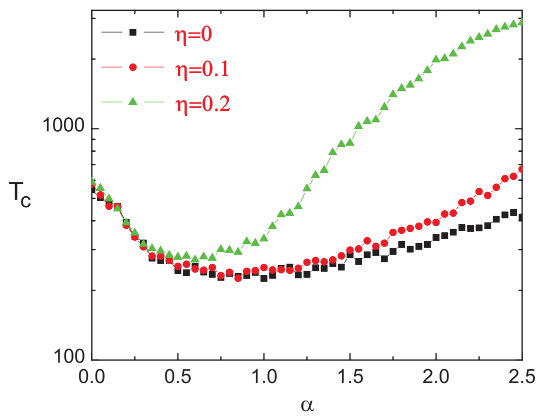

Fig. 6. The transient time step T_c as the function of α under low noisy environment. All the data points above are obtained by averaging over 500 different realizations. Here $R = 0.5$, $N = 800$ and $v = 0.04$.

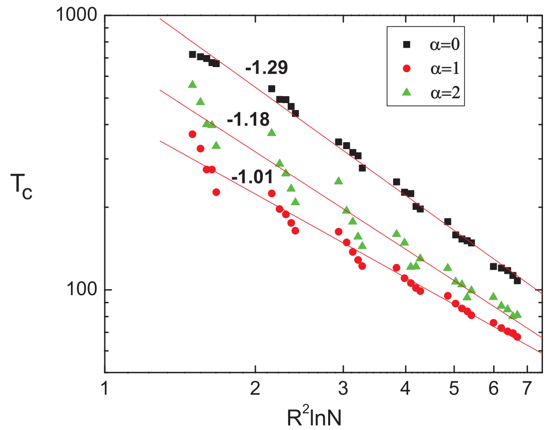

Fig. 7. The transient time step T_c as a function of $R^2 \ln N$ with different α. The data points can be well fitted linearly in double logarithmic coordinates even the $\alpha \neq 0$. Each data point is an average over 500 independent runs, and we consider the situation without noise. Here parameters R and N, respectively, vary from 0.5 to 1 (interval 0.1) and from 400 to 800 (interval 100).

(B) The optimal value is $\alpha = 1.0$ under different parameters for our model, i.e., the optimal phenomenon is robust to the parameters. However, in optimal view angle model, the optimal view angles are sensitive to the parameters (See the Fig. 3 in Ref. [19]).

The roles of the noise on the consensus of the swarm system are not considered in above cases, i.e., $\Delta\theta = 0$. However, the strength of the noise has significant effect on the transient time step T_c. In Fig. 6, we assume that $\Delta\theta$ is a random number chosen from the interval $[-\eta/2, \eta/2]$, and the strength of noise is controlled by parameter η. The result in Fig. 5 shows that the optimal phenomenon still exists when the strength of noise η isn't large.

For Vicsek model, we know that the transient time T_c ($\alpha = 0$ in our model) obeys a power function with $R^2 \ln N$, i.e. [13],

$$T_c \sim (R^2 \ln N)^k. \tag{6}$$

So we want to know whether such power-law behavior also exists in our model. From Fig. 7, one can see the transient time T_c also obeys a power function with $R^2 \ln N$, but the exponential parameter k for different values of α is somewhat different. More significantly, it is found that, compared with the classical Vicsek model, the convergence time is effectively shortened and the absolute slope is reduced, which demonstrates the virtue of this modified model in accelerating the convergence process.

4. Conclusion

The collective behavior of intelligent agents is not only a wide-spread phenomena in nature, but also an important problem requiring in-depth investigation in industrial engineering [13]. However, most of the previous studies assumed that each agent updates its direction by averaging all neighbors in its horizon scope. Given that the movement directions of intelligent agents may be easily affected by the agents in front of them when they are moving, we have studied a weighted collective model in this paper. In the weighted model, we use a tunable parameter α to characterize different degree of neighbors' impacts on the next direction of the center agent. The impacts of all neighbors are equally considered when $\alpha = 0$; otherwise $\alpha > 0$, the impacts of neighbors ahead are larger than the impacts of side neighbors or ones behind. By carrying out extensive simulation results on our proposed model we found that there exists an optimal $\alpha_{opt} \cong 1$ leading to the shortest the transient time T_c, i.e., the fastest consensus of self-driven system. Furthermore, we found that such optimal phenomenon is robust to different conditions, including the number of agents, the absolute velocity and rather small strength of noise, and so on.

Acknowledgments

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